MHD Casson non-Newtonian nanofluid over a nonlinear penetrable elongated sheet with thermal radiation and chemical reaction

Seethamahalakshmi VYAKARANAM¹, Venkata Ramana Reddy GURRAMPATI²* and Y Hari Krishna³

¹Department of Mathematics, PVP Siddartha Institute of Technology, Kanur, India - 520007.
²Department of Mathematics, Koneru Lakshmaiah Education Foundation, Vaddeswaram - 522502.
³Department of Mathematics, ANURAG Engineering College, Ananthgiri (v), Kodad, Suryapet, Telangana, India - 508206.

ABSTRACT
Consider a steady flow in two-dimensional of a viscous, incompressible Casson nano liquid over a nonlinear penetrable elongated sheet with radiation and chemical reaction. The Casson liquid rheological model is used to explain the non-Newtonian liquid attributes. Similarity variables are utilized to evaluate the governing flow model into set of nonlinear total differential equations. The outcomes of the flow equations were gotten by using Runge-Kutta alongside the shooting techniques. In other to explain the physics of the problem, impact of flow parameters are presented in graphs while computations on engineering curiosity are presented in table. Ahike in the Casson liquid term is observed to degenerate the fluid velocity alongside the momentum layer thickness. The impact of the imposed magnetic is felt by decreasing the velocity owning to the Lorentz force.

Keywords: Non-Newtonian fluid, elongated sheet, Lorentz force, thermal radiation, Chemical reaction.

Introduction

The thermal radiation acts a major role in industrial engineering. It major importance on heat transport process is in design of energy system utilized at a very higher temperature. Examples are missiles, gas turbines, space vehicles, nuclear power plants and satellites. Fagbade et al. [11] employed spectral approach to the problem of MHD flow of Walters-B liquid with thermal radiation alongside heat source.

www.psychologyandeducation.net

The consideration of heat alongside mass transport with MHD have received much attention by many scholars owning to their numerous applications in chemical engineering and sciences. Liquids such as ethylene glycol, water and oil are poor conductivity of heat, they are found very useful as a cooling materials for boosting manufacturing. The presence of MHD drags the flow owning to the production of Lorentz force. This indicate the usefulness of MHD in reducing the turbulence of fluid flow. 

Inspired by the literature discussed above to study a steady two-dimensional flow of a viscous, incompressible non-Newtonian nano liquid over a nonlinear penetrable elongated sheet with radiation and chemical reaction. Similarity transformation has been utilized to simplify the governing equations, and the reduced boundary value problems are solved using Runge-kutta method alongside shooting techniques.

Mathematical analysis

Consider steady, laminar, viscous and incompressible boundary layer flow of nano liquid past a nonlinear penetrable and stretchable surface. The $y$-coordinate is taken normal to the stretching surface while the flow occurs immediately $y \leq 0$. The Casson liquid flow arises owning to the stretching sheet that take place out of the slit at $x = 0$ and $y = 0$. Let us assume that a certain point at the plate, the speed is equivalent to the distance from the slit. Thus, the approximation of the boundary layer is valid on the flow. Starting from the wall where $U_x = ax^b$, the sheet is assumed.
to vary non-linearly with a distance of $x$. In the flow, $U_w = ax^n$, 'a' signifies constant, that is $a > 0$ while $n$ signifies nonlinear stretching term. According to the constitutive equation of Casson liquid as described as follows:

$$\tau_y = \begin{cases} \mu_y + \frac{P_c}{\sqrt{2\pi}} 2e_y & \text{when } \pi > \pi_c \\ \mu_y + \frac{P_c}{\sqrt{2\pi}} 2e_y & \text{when } \pi < \pi_c \end{cases}$$

(1)

where $P_c$ signifies Casson yield stress expressed as

$$P_c = \frac{\mu_0 \sqrt{2\pi}}{\beta}$$

(2)

$\mu_0 =$ plastic dynamic viscosity, $\pi = e_y e_y =$ multiplication of the component of deforming rate with itself and $e_y =$ deformation rate and $\pi_c =$ critical value subject to Casson model. The Casson liquid flow where $\pi > \pi_c$, we have

$$\mu_0 = \mu_0 + \frac{P_c}{\sqrt{2\pi}}$$

(3)

Putting equation (2) into (3), thus kinematic viscosity is subject to plastic dynamic viscosity $\mu_0$, the density $\rho$ and Casson term $\beta$ gives

$$\mu_0 = \frac{\mu_0}{\rho} \left(1 + \frac{1}{\beta}\right)$$

(4)

Putting into consideration all the assumptions, the flows equations that govern the model are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(5)

$$\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\mu}{\rho} \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_c^2 v^2}{\rho} u - \frac{\mu_0}{k \rho} \left(1 + \frac{1}{\beta}\right) u$$

(6)

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \left(\frac{\partial T}{\partial y}\right)^2 - \frac{1}{\rho c_p} \frac{\partial q}{\partial y} + \frac{\mu}{\rho c_p} \left(1 + \frac{1}{\beta}\right)$$

(7)

subject to:

$$u = U_w + S_1 \frac{\partial u}{\partial y} , v = \pm v_w , T = T_w + S_2 \frac{\partial T}{\partial y} , C = C_w + S_3 \frac{\partial C}{\partial y}$$

(8)

$$u \rightarrow 0 , T \rightarrow T_w , C \rightarrow C_w , \text{as, } y \rightarrow \infty$$

(9)

here $u$ and $v$ are velocity components in $x$ and $y$ direction, $v$ signifies viscosity, $\beta$ signifies Casson term, $\sigma$ signifies electrical conductivity, $B_c$ signifies magnetic constant, $\rho$ signifies density, $K$ signifies porosity term, $T$ signifies temperature, $\alpha$ signifies thermal conductivity, $\tau$ signifies heat capacity ratio, $D_a$ signifies mass diffusivity, $\mu$ signifies concentration, $D_t$ signifies thermophoretic diffusion coefficient, $T_w$ signifies ambient temperature, $\mu$ signifies coefficient of viscosity, $c_p$ signifies specific heat, $q_e$ signifies radiative heat flux, $k_e$ signifies chemical reaction term, $S_1, S_2, S_3$ and $S_5$ signifies slip terms respectively.

To simplify the radiative heat flux on the flow, application of Rosseland diffusion approximation is preference as reported in Alao et al.[25] and Fagbade et al.[27] such that:

$$q_e = \frac{4\sigma T^4}{3k_e}$$

(10)

From the above, the Stefan-Boltzman constant is $\sigma$, and the mean absorption coefficient is $k_e$. Let surmised that the difference in temperature in the flow are sufficiently small so that $T^4$ may be expressed in a linear form by expanding $T^4$ about
using Taylor series and neglecting higher terms to obtain
\[ T^4 - 4T^3 + 3T^4 \] (12)

substituting (12) into (11) and the result to the third term of the energy eqn to obtain
\[ \frac{\partial q_r}{\partial y} = \frac{16\sigma T_o^3}{3k_e} \frac{\partial^2 T}{\partial y^2} \] (13)

The similarity transformation defined as follows are employed in this study
\[ u = ax^\eta f'(\eta), v = -ax^{\eta-1} \sqrt{\frac{n+1}{n-1} f(\eta) + \frac{n-1}{n-2} f'(\eta)}, \eta = \frac{a^{\eta-1}}{v^{\eta-1}} y, \theta(\eta) = \frac{T-T_\infty}{T_o-T_\infty}, \phi(\eta) = \frac{C-C_o}{C_o-C_e} \] (14)

With respect to equation (14) above, the governing flow equations are simplified to obtain
\[
\left(1+\frac{1}{\beta}\right)f'''+\frac{n+1}{2}ff'-n(f')^2-\left(M+\frac{1}{\beta}\right)\frac{1}{K}f'=0
\] (15)

\[
\left(1+\frac{R}{Pr}\right)\theta''+\frac{n+1}{2}f\theta'-nf'\theta+Nb\theta\phi'+Nt(\theta')^2+PrEc \left(1+\frac{1}{\beta}\right)f''=0
\] (16)

\[
\phi''+\frac{n+1}{2}Lef\phi'+\frac{Ni}{Nb}\theta''-Kr\phi=0
\] (17)

with the constraints:

\[
f(0) = f_*, f'(0) = 1+L_1f''(0), \theta(0) = 1+L_2\theta'(0), \phi(0) = 1+L_3\phi'(0)
\] (18)

\[
f'(\infty) \to 0, \theta(\infty) \to 0, \phi(\infty) \to 0
\] (19)

Obviously, \( n \) signifies nonlinear stretching parameter while \( x \) signifies coordinate along the surface towards \( f_* \) and \( L_1 \). To simplify \( f_* \) as well as \( L_1 \) in a dimensionless form, \( P_m = \left(\frac{x^{n-1}(n+1)}{2}\right) \) is employed. Hence, \( f_* \) together with \( L_1 \) is subject to \( P_m \) as

\[
F_m = \frac{f_*}{\sqrt{P_m}}, L_1 = S\sqrt{P_m}
\]

The engineering curiosity are skin friction \( (C_f) \), local Nusselt \( (Nu) \) and Sherwood \( (Sh) \) defined as follows:

\[
C_f = \frac{\tau_w}{\rho u^2}, Nu = \frac{\chi q_w}{K(T_o-T_\infty)}, Sh = \frac{\chi q_m}{D_b(C_o-C_e)}
\]

and \( \tau_w \) signifies wall shear stress, \( q_w \) signifies heat flux while \( q_m \) signifies mass flux. They are

\[
\tau_w = \mu_b \left(1+\frac{1}{\beta}\right) \left(\frac{\partial u}{\partial y}\right)_{y=0}, q_w = -K \left(\frac{\partial T}{\partial y}\right)_{y=0}, q_m = -D_b \left(\frac{\partial C}{\partial y}\right)_{y=0}
\]

Employing equation (14) on the above to obtain

\[
Re^2C_f = \left(1+\frac{1}{\beta}\right)f''(0), Re^2Nu = -\theta'(0), Re^2Sh = -\phi(0)
\]

**Numerical approach**

The converted nonlinear differential Equations (15)–(17) with the boundary conditions (18) and (19) are elucidated by Runge-Kutta Fehlberg method along with shooting technique. This method has been proven to be adequate and gives accurate results for the boundary layer equations.

**Results and Discussion**

This section explains the outcomes of the results from solving equation (15)-(17) subject to (18) and (19) numerically. To describe the physics of the problem, results are presented in graphs. Figure 2 portrays the effect of porosity term \( K \) on the velocity plot. It is noticed that an incremental value of \( K \) leads to degeneration in the velocity plot as well as the momentum layer thickness. Physically, a hike in \( K \) makes very hard for the liquids particles to move freely and hereby resist the velocity gradually. Figure 3 portrays the behaviour of the imposed magnetic term \( M \) on the velocity plot. An increment in the value of \( M \) as shown in Figure 3 declines the velocity plot. Practically, imposing the magnetism in the flow direction of an electrically conducting liquid such as Casson produces Lorentz force. This force has
the tendency to drag the Casson flow by decreasing its velocity. Hence, the fluid velocity alongside its momentum layer thickness is declined. This implies that, increase in $M$ added more strength to the Lorentz force.

Figure 4 presents the impact of varying the nonlinear stretching term ($n$) on the velocity plot. Obviously from figure 4, an increment in the stretching term degenerates the velocity plot alongside the momentum layer thickness. Figure 5 portrays the impact of the Casson ($\beta$) electrically conducting liquid on the velocity plot. A large value of $\beta$ is detected to degenerate the velocity plot from 1 to 4. This indicates that the density alongside momentum layer thickness declines for a large value of $\beta$. Experimentally, when $\beta \to \infty$ it is detected that the behaviour of $\beta$ is equivalent to a Newtonian behaviour. Physically, the Casson liquid possesses a plastic dynamic viscosity originates a resistance to the liquid flow by decreasing the velocity plot.

Figure 6 portrays the role of thermal radiation ($R$) on the temperature plot. A large value of $R$ is observed to hike the temperature plot. This implies an enhancement in the thermal layer thickness. Physically, the thermal radiation helps to boost the thermal situation of the liquid environment. Hence, increasing $R$ helps to boost the heat on the liquid and hereby lead to enhancement in the temperature of the liquid. Figure 7 represents the impact of the Prandtl number ($Pr$) on the temperature plot. A less fluid temperature alongside thermal layer thickness is noticeable for a large Pr. Hence, when $Pr << 1$, it implies thermal diffusivity controls the flow behavior while implies momentum diffusivity controls the flow behavior. Therefore, is useful in controlling cooling rate of a conducting liquid electrically. In this study, there exist a wall temperature and free stream temperature which gives rise to $T_w - T_\infty$ with thermal boundary layer thickness.

Figure 8 shows the effect of Eckert number ($Ec$) on the temperature plot. An increment in the value of $Ec$ from 0.1 to 0.4 is observed to degenerate the temperature plot alongside the thermal layer thickness. $Ec$ portray the flow kinetic energy and its associated enthalpy. The viscous dissipation term (Eckert number) hike heat energy to the fluid environment. Physically, Eckert number and thermal radiation in a fluid flow increase the rate of heat transfer. Now, considering Eckert number, thermal radiation with thermally-stratified medium assist to boost the heat transport rate by elevating the thickness of thermal boundary layer. Figure 9 represents the effect of the thermophoresis ($Nt$) on the concentration plot. It is detected in figure 9 that a large value of $Nt$ hike the heat transport rate by enhancing the fluid temperature plot.

Figure 10 depicts the effect of the Brownian motion ($Nb$) on the concentration plot. A large value of $Nb$ is observed to declines the concentration plot as well as the solutal layer thickness. The result in figure 10 shows the impact of $Nb$ on concentration plot is opposite to that of $Nt$. Figure 11 shows the impact of the chemical reaction term ($Kr$) on the concentration plot. An incremental value of $R$ is observed to degenerate the concentration plot. Physically, it indicate a destructive chemical reaction. Figure 12 represents the effect of the Lewis number ($Le$) on the concentration plot. An increment in $Le$ is observed in figure 12 to degenerate the concentration plot. Figure 13 shows the slip effect on the velocity profile. The result in figure 13 shows a decrease in the velocity plot owning to a hike in the slip term. Figure 14 shows the slip effect on the temperature profile. The result in figure 14 shows a decrease in the temperature plot due to increase in the slip term. Figure 15 shows the slip effect on the species plot. The result in figure 15 shows a decrease in the concentration plot due to increase in the slip term. Table 1 shows the impact of various flow parameters on...
the engineering interest. All pertinent flow parameters were found to have great effect on the skin friction, Nusselt and Sherwood number.

**Figure 2**: Effect of $K$ on velocity profile

**Figure 3**: Effect of $M$ on velocity profile.

**Figure 4**: Effect of $n$ on velocity profile

**Figure 5**: Effect of $\beta$ on velocity profile.

**Figure 6**: Effect of $R$ on temperature profile.

**Figure 7**: Effect of $Pr$ on temperature profile
Figure 8: Effect of Ec on temperature profile

Figure 9: Effect of Nt on concentration profile

Figure 10: Effect of Nb on concentration profile.

Figure 11: Effect of Kr on concentration profile.

Figure 12: Effect of Le on concentration profile.

Figure 14: Effect of L_2 on temperature profile.
Figure 13: Effect of $L_1$ on velocity profile.

Figure 15: Effect of $L_3$ on temperature profile.

Table 1: Numerical computational value of the skin friction, Nusselt and Sherwood numbers

<table>
<thead>
<tr>
<th>M</th>
<th>Kr</th>
<th>n</th>
<th>B</th>
<th>R</th>
<th>Pr</th>
<th>Nt</th>
<th>Nb</th>
<th>Kr</th>
<th>Ec</th>
<th>Le</th>
<th>$L_1$</th>
<th>$L_2$</th>
<th>$L_3$</th>
<th>$f_w$</th>
<th>Cf</th>
<th>Nu</th>
<th>Sh</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>3</td>
<td>0.5</td>
<td>2</td>
<td>0.71</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.751056</td>
<td>0.576817</td>
<td>0.679085</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.803672</td>
<td>0.591244</td>
<td>0.679420</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.847451</td>
<td>0.607771</td>
<td>0.680427</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.884901</td>
<td>0.626931</td>
<td>0.682576</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3</td>
<td>0.5</td>
<td>2</td>
<td>0.71</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.826480</td>
<td>0.558263</td>
<td>0.679084</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.901764</td>
<td>0.583796</td>
<td>0.679305</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.932480</td>
<td>0.599216</td>
<td>0.679816</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
<td>2</td>
<td>0.71</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.719350</td>
<td>0.456873</td>
<td>0.576210</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.762119</td>
<td>0.541240</td>
<td>0.627048</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.803672</td>
<td>0.607771</td>
<td>0.680427</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>0.71</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.911101</td>
<td>0.456803</td>
<td>0.698083</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.990830</td>
<td>0.466261</td>
<td>0.709069</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.024158</td>
<td>0.484212</td>
<td>0.713204</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>3</td>
<td>0.5</td>
<td>1</td>
<td>0.71</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.803672</td>
<td>0.467836</td>
<td>0.611639</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.803672</td>
<td>0.525450</td>
<td>0.680427</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.803672</td>
<td>0.607771</td>
<td>0.721610</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>3</td>
<td>0.5</td>
<td>5</td>
<td>0.71</td>
<td>0.1</td>
<td>0.1</td>
<td>0.5</td>
<td>0.001</td>
<td>0.803672</td>
<td>0.374465</td>
<td>0.404055</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.803672</td>
<td>0.450143</td>
<td>0.582207</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.803672</td>
<td>0.796291</td>
<td>0.759728</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>3</td>
<td>0.5</td>
<td>5</td>
<td>0.71</td>
<td>0.1</td>
<td>0.1</td>
<td>0.5</td>
<td>0.001</td>
<td>0.803672</td>
<td>1.109419</td>
<td>0.797016</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.803672</td>
<td>0.371681</td>
<td>0.797016</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.803672</td>
<td>0.372770</td>
<td>0.869987</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Conclusion
The study at hand provides the numerical outcomes for a steady, viscous flow of Casson liquid alongside heat transport past an elongated sheet with thermal radiation and chemical reaction. The velocity decreases significantly owning to increase in Casson term. The temperature also degenerate significantly owning to increase in Prandtl number. The effect of increasing values of the magnetic term is to suppress the velocity, whereas the temperature is enhanced with increasing thermal radiation parameter. Prandtl number can be used to increase the rate of cooling in conducting flows.

References


[18] P R Kumari, D Sree Devi. Effect of radiation and radiation absorption on convective heat and mass transfer flow of


