Designing Controller Using Stable Krylov Subspace Method

Kumar Nikhil[1], Prof. (Dr.) Vijaya Laxmi[2], Prof. (Dr.) Dinesh Chandra[3]
[1]Assistant Professor, Birla Institute of Technology, Mesra, Ranchi, Patna Campus, Patna
[2]Professor, Birla Institute of Technology, Mesra, Ranchi
[3]Professor, Motilal Nehru National Institute of Technology, Allahabad (Prayagraj)
Email- [1]k.nikhil@bitmesra.ac.in, [2]vlaxmi@bitmesra.ac.in, [3]dinuchandra@rediffmail.com

ABSTRACT

In this work, a method for reduction of higher order system (HOS) into a low order model has been developed through modified Krylov subspace (MK-S) method, which is a combination of Routh-Padé approximation (RPA) method and Krylov subspace method. A low order controller has been then developed for this reduced order model (ROM). The controller is then implemented to the HOS and the performance of the HOS is evaluated. The comparison between responses of HOS and ROM has been done for some example problems and presented in the results. The results validate the efficacy of the designed low order controller for the higher order systems using proposed method.

Keywords

Controller, Model Reduction, Krylov subspace, Routh-Padé Approximation method.

Article Received: 10 August 2020, Revised: 25 October 2020, Accepted: 18 November 2020

Introduction

In the recent past, approximation of a higher order linear system by a low order model has received a substantial attention. A wide range of model order reduction techniques [1,2] have been proposed either in frequency domain or time domain in the recent past to reduce the order of a higher order system. Some of the frequency domain techniques are Padé approximation [3], Routh approximation [4-9], Routh-Padé approximation [10-11] and in time domain the techniques are like singular value decomposition (SVD) [12-14], balancing and truncation [12], Krylov subspace (K-S) [15-22, 32] methods. In the K-S method, Taylor series expansion is used to match some of the lower order coefficients of the original system transfer function and reduced order system transfer function. In this method, the matching of coefficients are done based on time moment matching and Markov moments matching[23]. When the matching is around $s = 0$, time moments matching is achieved and around $s = \infty$, Markov parameters matching is achieved. Krylov subspace is used to calculate projection matrices. After obtaining the projection matrices, these are used in the system matrices of the higher order system resulting in reduced order models. In this approach, the algorithms proposed by Lanczos [24], the Arnoldi [25-26] and two-sided Arnoldi [27-28] are used, which are numerically efficient methods. K-S method is also referred as method by projection in which projection is done from high dimensional space to the lower dimensional space and vice-versa using the concept of input and output subspaces. These days K-S method has become the best choice for reduction of order for very high order systems. However, the K-S method has a drawback that stability of the reduced order model is uncertain [29-33].

There are certain advantages of linear simple controllers over complex linear controllers such as less computational requirements. Hence, linear simple controllers are preferred. A brief review of different approaches for reduction of order of controller has been presented by Anderson and Liu in [34]. These approaches are divided into direct and indirect methods [35]. In direct methods, a low order controller is obtained directly [36-39] in which, usually a quadratic optimization problem is solved with an order constraint and a constraint of close loop stability. The indirect methods are of two types [40]. In the first type, a high order controller are derived from the original high order plants, by using some linear quadratic Gaussian (LQG) or H^{∞} design method, and then an approximation of the controller is determined. In the second type, a low order plant transfer function is computed from the original plant transfer function, and then a low order controller is designed to control the original plant.

In [34], a review of indirect strategies of first type have been presented and discussed using balanced realization [41-43], Hankel norm optimal approximation [34, 44-47], and q-covariance equivalent realization [48-50]. These techniques usually replace a stable high order model by other stable low order model which is not an optimal L_{∞} approximation. Further, in these techniques, no frequency weighting is generally employed [34]. However, in literature some frequency weighted versions are available for the first two methods in [51-53].

In this research work, first a higher order system (HOS) is approximated by using K-S method to reduce into a low order model (ROM). An indirect method of second type for reduction of order of controller is then used to design a low order controller for this ROM. The low order controller is then connected to the HOS and the performance of the HOS is evaluated and compared.

With the help of Routh-Padé approximation (RPA) method stability of the ROM is preserved [11,29]. The proposed method is termed as a modified Krylov subspace (MK-S) method. It combines RPA method to guarantee stability. In the first step, K-S method is used to derive the ROM. If the model turns out to be stable, the algorithm is terminated. If not, a next stable higher order model $G_k(s)(r+1 or r+2 or r+3...)$ is identified. Then RPA method is used to obtain a stable ROM as the second step. Then, a suitable controller is designed for the original as well as reduced order system. Thus, the proposed method completely or closely matches $2r$ terms (time moments/Markov parameters) of the original HOS and ROM and also preserves stability.

This paper has been organized as follows: For completion of presentation and clarity, the K-S method [18-22] and Routh-Padé approximation methods [11,29] have been reproduced in the Section 2. In Section 3, basic concepts of controller design have been presented. The proposed MK-S method has been explained in the Section 4. Section 5 deals with different numerical examples and their results. The last section comprises of the conclusion of proposed algorithm.

Krylov Subspace Method

In this section, the K-S method has been described. Consider a general single input single output (SISO) system [18,22], whose state model is given by

$$E\dot{x} = Ax + bu \quad (1)$$

$$y = c^T x \quad (2)$$

The input variable is *u* and output variable is *y*. The number of state variables *n* is the order of the system, where *E*, *A*, *b* and *c* represent the descriptor matrix, state matrix, input matrix and output matrix, respectively.

The transfer function $G_n(s)$ of the system in (1), (2) is described as

$$G_n(s) = c^T(sE - A)^{-1}b \quad (3)$$

$$= -c^T A^{-1}b - c^T(A^{-1}E)A^{-1}bs - c^T(A^{-1}E)^i A^{-1}bs^i \quad (4)$$

$$= c^T E^{-1}bs^{-1} + c^T(E^{-1}A)E^{-1}bs^{-2} + \ldots + c(E^{-1}A)^i E^{-1}bs^{-i} + \ldots \quad (5)$$

Equation (4) and (5) are the Taylor series expansions of the transfer function (3) around $s \to 0$ and $s \to \infty$, respectively.

It is desired to obtained a Krylov reduced order model of order r of the form

$$E_r\dot{x} = A_r x_r + b_r u \quad (6)$$

$$y = c_r^T x_r \quad (7)$$

for which

$$G_r(s) = c_r^T(sE_r - A_r)^{-1}b_r \quad (8)$$

Where, $E_r = W^T EV,\quad A_r = W^T AV,$

$b_r = W^T b,\quad c_r^T = c^T V,$

and E_r, A_r, b_r and c_r^T are obtained by applying projections from V and W to system (1) and (2). Matrices V and W are projection matrices. Thus, to determine a Krylov reduced order model, the projection matrices V and W which are basis of input Krylov subspace $K_{r1}(A^{-1}E, A^{-1}b)$ and output Krylov subspace $K_{r2}(A^{-T}E^T, A^{-T}c^T)$ respectively, chosen as

$$V = K_{r1}(A^{-1}E, A^{-1}b) = span\left\{A^{-1}b, \ldots \left(A^{-1}E\right)^{r-1}A^{-1}b\right\} \quad (9)$$

$$W = K_{r2}(A^{-T}E^T, A^{-T}c^T) = span\left\{A^{-T}c^T, \ldots \left(A^{-T}E^T\right)^{r-1}A^{-T}c^T\right\}$$

(10) with E, A, b, c^T from system (1), (2), with $A^{-T} = \left(A^{-1}\right)^T$.

By suitably changing the starting vectors $A^{-1}E, A^{-1}b$ in input Krylov subspace and $A^{-T}E^T, A^{-T}c^T$ in output Krylov subspace, some of the moments as well as some of the Markov parameters can be matched. The matrices V and W will be, $K_{r1}(A^{-1}E, (E^{-1}A)^{l_1} A^{-1}b)$ and $K_{r2}(A^{-T}E^T, (E^{-T}, A^T)^{l_2} A^{-T}c^T)$ respectively, both with rank r where $l_1, l_2 \in Z$ and $0 \le l_1, l_2 \le r$ then the first $2r - l_1 - l_2$ moments and the first $l_1 + l_2 = l$ Markov parameters of the system in (1), (2) and Krylov reduced order model will match [18].

While building up Krylov subspace, numerical problems may arise. To avoid this, classical Arnoldi [25] algorithm may be used, which finds a set of orthogonal vectors as a basis of a given Krylov subspace [22].

Routh-Padé Approximation Method

In this section, the Routh-Padé approximation method has been described. Let the transfer function $G_k(s)$ of a kth order system is represented as:

$$G_k(s) = \frac{a_1 s^{k-1} + a_2 s^{k-2} + \ldots + a_k}{s^k + d_1 s^{k-1} + \ldots + d_k} \quad (11)$$

$$= t_1 + t_2 s + \ldots + t_k s^{k-1} + \ldots \quad (12)$$

$$= M_1 s^{-1} + M_2 s^{-2} + \ldots + M_k s^{-k} + \ldots \quad (13)$$

Equation (12) and (13) are Taylor series expansions of this transfer function around $s \to 0$ and $s \to \infty$, respectively. The coefficients of series in (12) and (13) are called time moments and Markov parameters, respectively.

It is desired to obtained a ROM of order r $(r < k)$ as

$$G_{rk}(s) = \frac{a_1' s^{r-1} + a_2' s^{r-2} + \ldots + a_r'}{s^r + d_1' s^{r-1} + \ldots + d_r'} \quad (14)$$

$$= t_1' + t_2' s + \ldots + t_r' s^{r-1} + \ldots \quad (15)$$

$$= M_1' s^{-1} + M_2' s^{-2} + \ldots + M_r' s^{-r} + \ldots \quad (16)$$

The RPA method is basically a nonlinear optimization procedure in which not only first r terms (time moments/ Markov parameters) are fully retained but errors between the subsequent time moments /Markov parameters are also minimized preserving stability.

The formulation of the objective function has been explained, assuming r even. For this, it can be verified that, when the following equations hold true

$$a'_{r+1-i} = \sum_{j=1}^{i} t'_j d'_{r-i+j}, \qquad a'_i = \sum_{j=1}^{i} M'_j d'_{i-j} \text{ for } i = 1, \ldots r/2 \quad (17)$$

$$\left.\begin{aligned} t'_i &= \left(-\sum_{j=1}^{i-1} t'_j d_{r-i+j} + \sum_{j=1}^{r+1-i} M'_j d'_{r+1-i-j}\right) d'^{-1}_r \\ M'_i &= \sum_{j=1}^{r+1-i} t'_j d_{i-1+j} - \sum_{j=1}^{i-1} M'_j d'_{i-j} \end{aligned}\right\} \quad (18)$$

for $i = \frac{r}{2} + 1, \frac{r}{2} + 2, \ldots$

where, $d'_0 = 1; d'_i = 0$ for $i \notin \{0, \ldots, r\}; t'_i, M'_i = 0$ for $i < 1$.

A stable model for which r equations given by

$$t'_i - t_i = 0, \quad M'_i - M_i = 0, \text{ for } i = 1, \ldots r/2 \quad (19)$$

are satisfied is sought, with (19), equation (17) becomes

$$a'_{r+1-i} = \sum_{j=1}^{i} t_j d'_{r-i+j},\quad a'_i = \sum_{j=1}^{i} M_j d'_{i-j},\ \text{for } i = 1,\ldots r/2 \qquad (20)$$

There may be infinite number of stable models for which equation (20) is satisfied $[8,23]$. To exploit this arbitrariness, an objective function Z is constructed to minimize a weighted squared sum of the errors of matching time moments and Markov parameters of the system with those of the model, i.e., to minimize Z

$$Z = \sum_{i=1}^{r/2} \alpha_i \left(1 - \frac{t'_{r/2+i}}{t_{r/2+i}}\right)^2 + \sum_{i=1}^{r/2} \beta_i \left(1 - \frac{M'_{r/2+i}}{M_{r/2+i}}\right)^2 \qquad (21a)$$

where $\alpha_i's$,and $\beta_i's$ are nonnegative numbers. It may be noted that $(21a)$ can be adjusted if only time moments are considered as

$$Z = \sum_{i=1}^{r} \alpha_i \left(1 - \frac{t'_{r+i}}{t_{r+i}}\right)^2 \qquad (21b)$$

and if only Markov parameters are considered $(21a)$ takes the following form:

$$Z = \sum_{i=1}^{r} \beta_i \left(1 - \frac{M'_{r+i}}{M_{r+i}}\right)^2 \qquad (21c)$$

Using (18) subject to (19), the objective function Z can be expressed as

$$Z = f\left(d'_1,\ldots,d'_r\right) \qquad (22)$$

The formulation for r being odd can be done in a similar manner. For stability preservation, following $[30]$, the denominator coefficients of (14) can be expressed as

$$d'_1 = b'_1,\quad d'_2 = b'_2 + b'_3 + \ldots + b'_r,\ d'_3 = b'_1\left(b'_3 + b'_4 + \ldots + b'_r\right)\ldots,$$
$$d'_r = b'_{1+\eta} b'_{3+\eta} \ldots b'_{r-2} b'_r, \qquad (23)$$

where, $\eta = 1$ for even r and $\eta = 0$ for odd r, for a given r, (23) can be written by constructing a Routh array which has, in its first column, the entries $[30]$

$$1, b'_1, b'_2, b'_1 b'_3, b'_2 b'_4, b'_1 b'_3 b'_5, \ldots, b'_{1+\eta} b'_{3+\eta} \ldots b'_{r-2} b'_r \qquad (24)$$

Comparing the entries of the first row of this array with $1, d'_2, d'_4, \ldots$ and those of the second row with $d'_1, d'_3, \ldots$ yields (23). For example, for $r = 4$, (23) becomes

$$d'_1 = b'_1,\ d'_2 = b'_2 + b'_3 + b'_4,\ d'_3 = b'_1\left(b'_3 + b'_4\right),\ d'_4 = b'_2 b'_4, \qquad (25)$$

The necessary and sufficient condition for all the poles of (14) to be strictly in the left-half plane $[30]$ is

$$b'_1 > 0,\ldots,b'_r > 0\ \left(\Rightarrow d'_1 > 0,\ldots,d'_r > 0\right) \qquad (26)$$

The problem is to minimize (22) subject to (23) and (26).

Application of Luus and Jaakola Algorithm

The algorithm of Luus Jaakola (LJ) is developed for optimization purpose $[31]$. It is called LJ optimization method. This algorithm is found to be suitable for solving above stated problem. Although, the algorithm searches local minima, in the examples considered in this work, it has converged to improved approximants.

Design of Controller

Consider the control system $[54]$ as shown in Fig.3.1. Given $G_n(s)$ and $H(s)$, the problem is to derive the transfer function of the controller $C(s)$ which yields the desired response of the closed loop system.
A classical approach to the design of the controller $C(s)$ is to specify the desired (also called reference) closed loop transfer function $T(s)$, equate it to the closed loop transfer function, and solve for the controller $[54]$. Thus

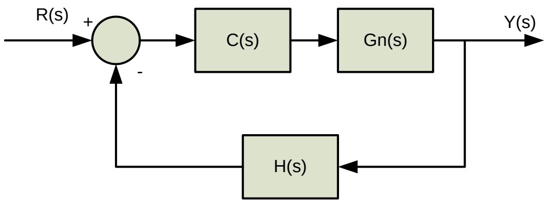


Fig. 3.1. Control Configuration

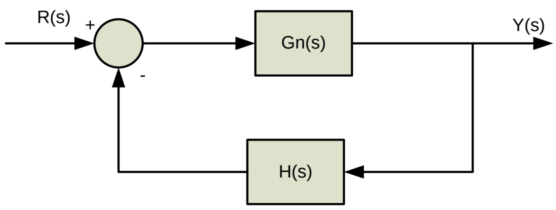
$$T(s) = \frac{G_n(s)C(s)}{1 + G_n(s)C(s)H(s)} \qquad (27)$$

On simplification for controller, (27) yields

$$C(s) = \frac{T(s)}{G_n(s)\left[1 - T(s)H(s)\right]}$$

$$= \frac{TN(s)G_n D(s)HD(s)}{G_n N(s)\left[TD(s)HD(s) - TN(s)HN(s)\right]} \qquad (28)$$

Where TN(.) and TD(.) indicate numerator and denominator respectively.
Now consider the closed loop system shown in Fig. 3.2. By approximating $G_n(s)$ by a reduced order model transfer function $G_{rk}(s)$, Fig.3.3 is obtained. In other words, the system of Fig.3.2 is approximated by that of Fig.3.3, where $H(s)$ is assumed to be same in both these figures. Pertaining to these two systems, the following result was previously arrived at $[55]$



.

Fig. 3.2. A closed loop system

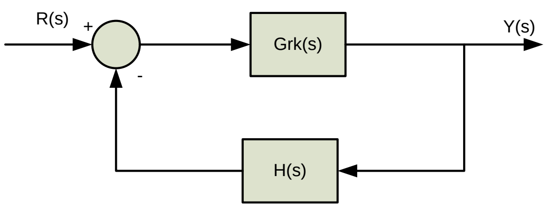


Fig. 3.3. A reduced order approximant of the system of Fig. 3.2

Modified Krylov Subspace (Mk-S) Algorithm

The proposed MK-S algorithm has been described as follows.

Step 1. Set the order of the reduced order model r for the given n^{th} order system $G_n(s)$. Choose l_1 and l_2 such that $l_1 + l_2 = l$ Markov parameters and $2r - l_1 - l_2$ time moments of the HOS and ROM match.

Step 2. Find the reduced order (r^{th} order) model $G_r(s)$ from (8).

Step 3. If $G_r(s)$ is stable then terminate the process otherwise go to next step.

Step 4. Find the next stable higher Krylov reduced order model $G_k(s)$, from Equation (8), where $k = (r+1) or (r+2) or ... (n-1)$

Step 5. Obtain the r^{th} order model $G_{rk}(s)$ from $G_k(s)$ using (14).

Step 6. Then the controller is designed (As per section

Numerical Examples

The step by step procedure to design of a controller has been explained in this section with the help of examples as presented below.

Example 1: Consider the following stable 4^{th} order system having transfer function

$$G_4(s) = \frac{s^3 + 12s^2 + 54s + 72}{s^4 + 18s^3 + 97s^2 + 180s + 100} \tag{29}$$

Derive a reduced order $(r=2)$ model of (29). Using the method described in the above section. The second order approximant to (29) turns out to be

$$G_{23}(s) = \frac{s + 24.84}{s^2 + 37.235s + 34.5} \tag{30}$$

Choose a reference model which satisfies the control specifications. In this example, a standard second order transfer function is chosen with damping ratio $\xi = 0.7$ and natural frequency $\omega_n = 1.5 rad/s$. Thus

$$T(s) = \frac{2.25}{s^2 + 2.1s + 2.25} \tag{31}$$

Derive the reduced order controller from (28) as

$$C_2(s) = \frac{C_2N(s)}{C_2D(s)} = \frac{TN(s)G_2D(s)HD(s)}{G_2N(s)[TD(s)HD(s) - TN(s)HN(s)]} \tag{32}$$

Here,

$$TN(s) = 2.25$$

$$TD(s) = s^2 + 2.1s + 2.25$$

$$G_2N(s) = s + 24.84$$

$$G_2D(s) = s^2 + 37.235s + 34.5$$

and $HN(s) = HD(s) = 1$. Therefore, $C_2(s)$ is obtained as

$$C_2(s) = \frac{2.25s^2 + 83.77875s + 77.625}{s^3 + 26.94s^2 + 54.414s} \tag{33}$$

The transfer function, $O(s)$, of the system (see Fig.3.1) with reduced order controller $C_2(s)$ takes the form

$$O(s) = \frac{C_2(s)G(s)}{1 + C_2(s)G(s)H(s)}$$

$$O(s) = \frac{2.25s^5 + 110.77875s^4 + 387.6271875s^3 + 5617.5525s^2 + 10223.82s + 5589}{s^7 + 44.94s^6 + 638.584s^5 + 3937.05075s^4 + 11580.505188s^3 + 23309.1525s^2 + 25320.42s + 10953} \tag{34}$$

the step responses of (31) and (34) are shown in Fig. 5.1. It is seen that the response of the system with reduced-order controller is satisfactory. Fig. 5.1 also depicts the responses of (29) and (30). Clearly (30) is a good approximant (Krylov Routh Pade` approximant) to (29).

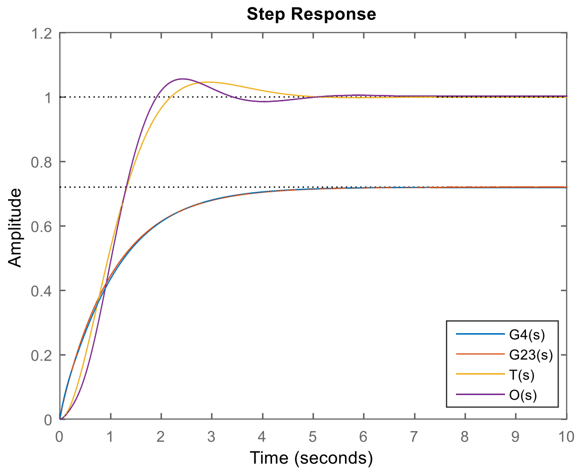


Fig. 5.1: Comparison of Step responses for original system $G_4(S)$, ROM $G_{23}(s)$, T(s) and O(s)

Example 2: Consider the following stable 6^{th} order System having transfer function

$$G_6(s) = \frac{2s^5 + 70s^4 + 762s^3 + 3610s^2 + 7700s + 2400}{s^6 + 41s^5 + 571s^4 + 3491s^3 + 10060s^2 + 13100s + 6000} \tag{35}$$

Derive a reduced order $(r=2)$ model of (35) .using the method described in the above section. The second order approximant to (35) turns out to be

$$G_{23}(s) = \frac{1.297s + 0.4997}{s^2 + 1.962s + 1.249} \tag{36}$$

Choose a reference model which satisfies the control specifications. In this example, a standard second order transfer function is chosen with damping ratio $\xi = 0.7$ and natural frequency $\omega_n = 1.5 rad/s$. Thus

$$T(s) = \frac{2.25}{s^2 + 2.1s + 2.25} \tag{37}$$

Derive the reduced order controller from (28) as

$$C_2(s) = \frac{C_2N(s)}{C_2D(s)} = \frac{TN(s)G_2D(s)HD(s)}{G_2N(s)[TD(s)HD(s) - TN(s)HN(s)]} \tag{38}$$

Here,

$$TN(s) = 2.25$$
$$TD(s) = s^2 + 2.1s + 2.25$$
$$G_2N(s) = s + 24.84$$
$$G_2D(s) = s^2 + 37.235s + 34.5$$

and $HN(s) = HD(s) = 1$. Therefore, $C_2(s)$ is obtained as

$$C_2(s) = \frac{2.25s^2 + 4.4145s + 2.81025}{1.297s^3 + 3.2234s^2 + 1.04937s} \quad (39)$$

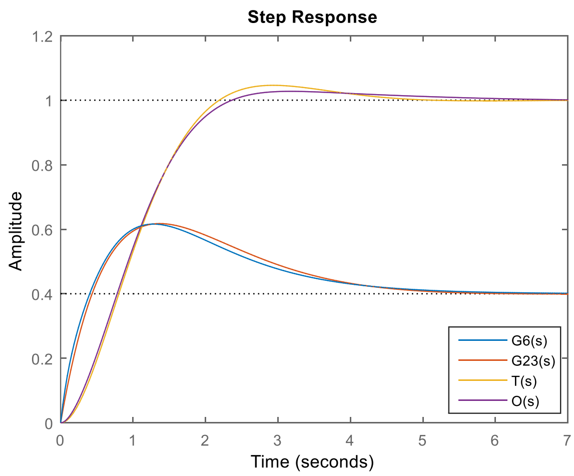


Fig. 5.2: Comparison of Step responses for original system $G_6(S)$, ROM $G_{23}(s)$, T(s) and O(s)

The transfer function, $O(s)$, of the system (see Fig.3.1) with reduced order controller $C_2(s)$ takes the form

$$O(s) = \frac{C_2(s)G(s)}{1 + C_2(s)G(s)H(s)}$$

$$O(s) = \frac{4.5s^7 + 166.329s^6 + 2029.1355s^5 + 11683.0665s^4 + 35402.7555s^3 + 49536.6525s^2 + 32233.725s + 6744.8}{1.297s^9 + 56.4004s^8 + 878.29577s^7 + 6577.74157s^6 + 26929.03517s^5 + 64764.52117s^4 + 95967.9577s^3 + 82623.7995s^2 + 38529.945s + 6744.8} \quad (40)$$

the step responses of (37) and (40) are shown in Fig. 5.2. It is seen that the response of the system with reduced-order controller is satisfactory. Fig. 5.2 also depicts the responses of (35) and (36). Clearly (36) is a good approximant (Krylov Routh Pade` approximant) to (35).

Conclusion

In this work, a low order controller is designed using combination of Krylov and Routh Padé approximation method and termed as modified Krylov subspace method. The classical Krylov subspace method has the shortcoming of producing unstable reduced order models for a stable higher order system. In the proposed method, the stability is preserved in the reduced order model by making use of Routh Padé approximation technique. Once the HOS is reduced the controller is developed for this ROM and then finally implemented to HOS. The results corroborate the feasible and effective use of the designed controller using proposed method.

References

[1] Fortuna, L., Nunnari, G. and Gallo, A., "Model order reduction techniques with application in electrical engineering", London, UK; Springer-Verlag, 1992.

[2] Peter Benner, Serkan Gugercin, Karen willcox, “A survey of projection based model reduction method for parametric dynamical systems” SAM REVIEW, Vol-57, No.-4 pp. 483-531, 2015.

[3] J. Pal,"Stable reduced-order Pade approximants using the Routh-Hurwitz array ", Electronic Letters, Vol. 15 No, 8,12th April 1979.

[4] M.F.Hutton and Bernard Friendland, “Routh approximation for Reducing Order of Linear, Time-Invariant System", IEEE Trans. on Automat. Contr., Vol.ac-20, No.3, June 1975.

[5] Y. Shamash, “Failure of the Routh Hurwitz method of reduction”, IEEE Trans. Automat. Contr., Vol. AC-25, pp. 313-314, 1980.

[6] V. Singh, “Improved stable approximants using the Routh array,” IEEE Trans. Automat. Contr., Vol. AC-26, pp. 581–582, 1981.

[7] Rao. A. S, “On Routh approximation”, Proc. IEEE, 272-274, 1983.

[8] Singh, V., “Nonuniqueness of model reduction using the Routh approach”, IEEE Trans. Automat. Contr., 24, 650-651, 1979.

[9] V. Krishnamurthy and V. Seshadri, "Model reduction using the Routh stability criterion", IEEE Trans. Automat. Contr., Vol. AC-23, pp. 729-731, Aug. 1978.

[10] Y. Shamash. “Model reduction using the Routh stability criterion and the Pade approximation technique,” Int. J. Contr., Vol. 21, no. 3, Mar. 1975.

[11] V.Singh, D.Chandra and H. Kar, “Improved Routh – Pade Approximants: A Computer Aided Approach”, IEEE Trans.

Automat. Contr., vol.49, no.2, pp.292-296, 2004.

[12] B. C. Moore. Principal Component Analysis in Linear Systems: Controllability, Observability and Model Reduction. IEEE Trans. Aut. Control, 26:17–31, 1981.

[13] Antoulas, A.C.: ‘Approximation of Large-Scale Dynamical Systems’ (Society for Industrial and Applied Mathematics, Philadelphia, 2005).

[14] K. Glover. All optimal Hankel-Norm Approximations of Linear Multivariable Systems and Their Error Bounds. International Journal of Control, 39:1115–1193, 1984

[15] Marek Rydel, Włodzimierz Stanisławski, “Selection of reduction parameters of Rational Krylov Methods for complex MIMO LTI models using evolutionary algorithm”, IEEE International Conference on Methods and Models in Automation and Robotics, Sep., 2014.

[16] Heiko K. F. Panzer, Stefan Jaensch, Thomas Wolf and Boris Lohmann, “A Greedy Rational Krylov Method for H2-Pseudooptimal Model Order Reduction with Preservation of Stability”, American Control Conference Washington, D.C, USA, June, 2013.

[17] Jianyong Xie and Madhavan Swaminathan, “System-Level Thermal Modeling Using Nonconformal Domain Decomposition and Model-Order Reduction”, IEEE Transaction on Components, Packaging and Manufacturing Technology, Vol.-4, No-1, Jan. 2014.

[18] Salimbahrami, B. and Lohmann, B. “Krylov Subspace Methods in Linear Model Order Reduction: Introduction and Invariance Properties”, Scientific Report, Aug, 2002.

[19] Heiko K. F. Panzer, Thomas Wolf, Boris Lohmann, “H2 and H∞ Error Bounds for Model Order Reduction of Second Order Systems by Krylov Subspace Methods”, European Control Conference, Zürich, Switzerland, July, 2013.

[20] Freund, R. W. “Passive Reduced-Order Modeling via Krylov Subspace Methods”. IEEE International Symposium on Computer Aided Control System Design, Sep., 2000.

[21] Grimme, E. J. “Krylov Projection Methods for Model Reduction”. PhD thesis, Dept. of Electrical Engineering, University of Illinois at Urbana Champaign, 1997.

[22] Boris Lohmann and Behnam Salimbahrami, “Introduction to Krylov subspace Methods in Model order Reductuion”, Methods and Application in Automation, pp. 1-13, Aachen, Shaker Verlag, 2003.

[23] Singh, V., “Remarks on system simplification using the Routh stability array”, IEEE Proc., 69, p. 662, 1981.

[24] C. Lanczos. An Iteration Method for the Solution of the Eigenvalue Problem of Linear Differential and Integral Operators. J. Res. Nat. Bureau Stan., 45, pp. 255–282 , 1950.

[25] W. E. Arnoldi, “The principle of minimized iterations in the solution of the matrix eigenvalue problem”, Quart. Appl. Math., 9, pp. 17–29, 1951.

[26] Mohamed Kouki , Mehdi Abbes, Abdelkader Mami, “Arnoldi Model Reduction for Switched Linear systems”, IEEE International Conference on Modeling Simulation and Applied Optimization, April, 2013.

[27] B. Salimbahrami, B. Lohmann, T. Bechtold, and J. Korvink,”A Two-Sided Arnoldi- Algorithm with Stopping Criterion and an application in Order Reduction of MEMS”, Mathematical and Computer Modelling of Dynamical Systems, 11(1), pp. 79–93, 2005.

[28] J. Cullum and T. Zhang, “Two-sided Arnoldi and Non-symmetric Lanczos Algorithms”, SIAM Journal on Matrix Analysis and Applications, 24(2), pp. 303–319, 2002.

[29] V. Singh, "Obtaining Routh-Pade approximants using the Luus-Jaakola algorithm," IEEE Proc. Control Theory Appl., vol. 152 No., Mar., 2005.

[30] Parks, P. C, "A new proof of the Routh-Hurwitz criterion using the second method of Lyapunov", Proc. Camb. Philos. Soc., pp. 694-702, 1962,

[31] Luus, R., and Jaakola T.H.I, "Direct search and systematic reduction of size of search region",AIChE Journal, 19, pp. 760-766, 1973.

[32] N.Kumar,K.J.Vinoy,S. Gopalakrishnan, "A reduced order model for electromagnetic scattering using multilevel Krylov subspace splitting", ICCEM, pp. 341-343, 2015.

[33] A.Narain, D. Chandra and K. S. Ravindra, "Model order reduction of discrete-time systems using Fuzzy C- Means clustering." Int. Journal of computer, Electrical, Automation, control and information Engineering. 7(7), PP. 1008-1015, 2013.

[34] Anderson, B.D.O., and Liu, Y., "Controller reduction: concepts and approaches", IEEE Trans. Automat. Contr., vol. 34, PP. 802-812, 1989.

[35] Hyland, C.C., and Richter S., "On direct versus indirect methods for reduced order controller design", IEEE Trans. Automat. Contr., vol. 35, PP. 377-379, 1990.

[36] Bernstein, D.S., and Hyland, D.C., "The optimal projection equations for fixed order dynamic compensation", IEEE Trans. Automat. Contr., vol. AC-29, PP. 1034-1037, 1985.

[37] Bernstein, D.S., and Hyland, D.C., "Optimal projection/ maximum entropy stochastic modelling and reduced order design synthesis", Proc. IFAC workshop on Model errors and compensation, Boston, MA, 1985.

[38] Gangsaas, D., Bruce, K.R,. Blight, J.D,. and Ly, U.L,. "Application of modern synthesis to aircraft control: three case studies", IEEE Trans. Automat. Contr., vol. AC-31, PP. 995-1104, 1986.

[39] Hyland, D.C., and Bernstein, D.S., "The optimal projection equations for model reduction and the relationships among the methods of Wilson, Skelton, and Moore", IEEE Trans. Automat. Contr., vol. AC-30, PP. 1201-1211, 1985.

[40] Paraskevopoulos, P.N., "Exact transfer function design using output feedback", IEE Proc., vol. 123, PP. 831-834, 1976.

[41] Moore, B.C., "Principal component analysis in linear system: controllability, observability and model reduction", IEEE Trans. Automat. Contr., vol. AC-26, PP. 17-32, 1981.

[42] Pernebo, L., and Silverman, L.M., "Model reduction via balanced state space representation", Proc. 18th Conf. Decision Contr., Lauderavle, F.L, PP. 12-14, Dec.1979.

[43] Pernebo, L., and Silverman, L.M., "Model reduction via balanced state space representation", IEEE Trans. Automat. Contr., vol. AC-27, PP. 382-387, 1982.

[44] Glover, K., "All optimal Hankel norm approximations of linear multivariable systems and their $L\infty$ error bounds" , Int. J. Contr., vol. 39, PP. 1115-1193, 1984.

[45] Glover, K., and Limebeer, D.J.N., "Robust multivariable control system design using optimal reduced order plant methods", Proc. ACC, San Francisco, P. 664, 1983.

[46] Kung, S.Y., and Lin, D.W., "Optimal Hankel-norm model reductions: multivariable system", IEEE Trans. Automat. Contr., vol. AC-26, PP. 832-852, 1981.

[47] Kung, S., and Lin, D.W., "A state space formulation for optimal Hankel-norm approximations", IEEE Trans. Automat. Contr., vol. AC-26, PP. 942-946, 1981.

[48] Anderson, B.D.O., and Skelton, R.E., "The generation of all q-Markov covers", IEEE Trans. Circuits Syst., vol. CAS-35, PP. 375-384, 1984.

[49] Yousuff, A., and Skelton, R.E "Covariance equivalent realization with application to

model reduction of large scale systems", Contr. Dynamic Syst., vol.22, PP.273-348, 1985.

[50] Yousuff, A., Wagie, D.A., and Skelton, R.E "Linear system approximation via covariance equivalent realization", J. Math. Analy. Appl., vol.106, PP.91-115, 1985.

[51] Anderson, B.D.O., "Weighted Hankel-norm approximation: calculation of bounds", Syst. Contr. Lett., vol.7, PP. 247-255, 1986.

[52] Enns, D., "Model reduction with balanced realizations: an error bound and a frequency weighted generalization", Proc. 23rd IEEE Conf. Decis. Contr., Las Vegas, PP. 127-132, 1984.

[53] Latham. G., and Ander, B.D.O, "Frequency weighted optimum Hankel-norm approximation of stable transfer function," Syst. Contr. Lett., vol. 5, PP. 229-236, 1985.

[54] Aguirre, L.A., "Designing controllers by means of model reduction techniques", Electron. Lett., vol.29, PP. 389-390, 1993.

[55] Shamash, Y., "Closed loop feedback systems using reduced order models", Electron. Lett., vol.12, PP. 638-639, 1976.