

Pulsing Flows of a Viscous Incompressible Liquid in a Pipe with Elastic Walls

Navruzov Kuralbay¹, Begjanov Amirbek Shixnazarovich², Khujatov Nurbek Jumaboyevich³

¹Professor, Urgench state university

²PhD student, Urgench state university

³Teacher, Urgench state university

Abstract: As you know, the recent intensive introduction into practice of flexible pipelines made of polymer synthetic materials, pulsating fluid flow in elastic pipes is of great importance. As you know, the recent intensive introduction into practice of flexible pipelines made of polymer synthetic materials, pulsating fluid flow in elastic pipes is of great importance. By solving the problem, the necessary hydrodynamic parameters will be determined, such as pressure distributions, velocities, flow rates, the speed of propagation of the pulse wave pressure and their decay. For the first time in this article, a decrease in hydraulic resistance in a pulsating flow through pipes due to the elasticity of the wall will be determined. The dependence of the dimensionless value of the pressure pulse wave on the vibrational number

was investigated α . The speed of the pulse wave was compared with the speed of Moens-Korteweg C_∞ , and significant differences were revealed between them occurring at lower values of the Womersley oscillatory parameter, at large values of which significant differences are not observed. The dependence of the reciprocal damping per wavelength on the vibrational number α , was also investigated; it was shown that the damping is free at smaller values of the Womersley vibrational parameter, practically equal to zero, and at large values of which it asymptotically approaches unity.

Keywords: Elastic modulus, Moens-Korteweg, elastic modulus, flat, permeability, hydraulic resistance, longitudinal speed, lateral speed, pressure, density, vibration amplitudes, circular vibration frequency, harmonic number.

Article Received: 16th October, 2020; Article Revised: 30th December, 2020; Article Accepted: 08th January, 2021

Introduction

In recent years, the intensive introduction into practice of flexible pipelines made of polymer synthetic materials is of great importance [1-3], pulsating fluid flows in elastic pipes. Also in this area, pulsating fluid flows in pipes are of no small importance, taking into account the various mechanical properties of the wall [7-17]. Based on this consideration, this article will investigate the pulsating flow of a viscous fluid in an elastic tube. By solving the problem, the necessary hydrodynamic parameters will be determined, such as pressure distributions, velocities, flow rates, the speed of propagation of the pulse wave pressure and their attenuation. For the first time in this article, a decrease in hydraulic resistance in a pulsating flow through pipes due to the elasticity of the wall will be determined.

Problem statement and solution methods

Let us formulate a simplified problem, which is of no small importance in studies of the pulsating flow of a viscous fluid in pipes with elastic walls [4-8]. For this, we assume that the relative amplitude of the wall deformation to the radius is too small

compared to unity, i.e. $\frac{\Delta R}{R} \ll 1$. And also

the fluid flow occurs in a long pipeline, so

that $\varepsilon = \frac{R}{L} \ll 1$. Then, neglecting small

quantities, from the system of equations for the flow of a viscous fluid we have

$$\begin{cases} \frac{\partial v_x}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + v \left(\frac{\partial^2 v_x}{\partial r^2} + \frac{1}{r} \frac{\partial v_x}{\partial r} \right), \\ \frac{\partial p}{\partial r} = 0, \quad \frac{\partial v_x}{\partial x} + \frac{\partial v_r}{\partial r} + \frac{v_r}{r} = 0. \end{cases} \quad (1)$$

To deform the pipeline wall on the basis of the accepted assumption at small wall deformations, it is sufficient to use the Lightfoot equation [3]

$$\rho_\omega h \frac{\partial^2 u_r}{\partial t^2} = (p - p_c) - \frac{E h u_r}{R^2 (1 - \nu_1^2)}, \quad (2)$$

u_r – the ratio of radial deformation ΔR to the radius of the pipe at rest;

p_c – ambient pressure; ρ_ω – pipe wall density; h – wall thickness; E – elastic modulus; R – the radius of the middle surface of the pipe wall; ν_1 – poisson’s ratio.

The left side of the equation expresses the inertia of the pipe wall, however, they are negligible, so we neglect them. Then (2) has the form

$$p - p_c = \frac{E h u_r}{R^2 (1 - \nu_1^2)}. \quad (3)$$

Note that the adhesion of the liquid and the permeability of the pipe wall are determined by the boundary conditions for the velocity components:

$$v_x = 0, \quad v_r = \frac{\partial u_r}{\partial t} \text{ at } r = R. \quad (4)$$

If the wall deformation is small, then we can assume that

$$v_r \Big|_{r=R+u} = v_r \Big|_{r=R}. \quad (5)$$

Differentiating equations (3) with respect to the variable t, taking into account (5), we write

$$\frac{\partial \bar{p}}{\partial t} = \frac{E h g_{r=R}}{R^2 (1 - \nu_1^2)} \quad (6)$$

where $\bar{p} = p - p_c$.

By integrating the continuity equation from 0 to R, we find

$$\frac{\partial \bar{V}_x}{\partial x} = -\frac{2}{R} g_{r=R}, \quad (7)$$

where \bar{V}_x –average flow rate.

Then the relationship between pressure and average velocity is described by the equation

$$\frac{\partial \bar{p}}{\partial t} = -\frac{\bar{E} h}{2R} \frac{\partial \bar{V}_x}{\partial x} \text{ or } \frac{\partial \bar{p}}{\partial t} = -\frac{\bar{E} h}{2R} \frac{\partial \bar{V}_x}{\partial x}, \quad (8)$$

where $\bar{p} = p - p_c$, $\bar{E} = \frac{E}{1 - \nu_1^2}$.

Thus, the simplified system of equations of motion of a viscous fluid in pipes with elastic walls will take the final form:

$$\begin{cases} \frac{\partial V_x}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + v \left(\frac{\partial^2 V_x}{\partial r^2} + \frac{1}{r} \frac{\partial V_x}{\partial r} \right), \\ \frac{\partial p}{\partial r} = 0, \quad \frac{\partial V_x}{\partial x} + \frac{\partial V_r}{\partial r} + \frac{V_r}{r} = 0, \\ \frac{\partial \bar{p}}{\partial t} = -\frac{\bar{E}h}{2R} \frac{\partial \bar{V}_x}{\partial x}. \end{cases} \quad (9)$$

To solve a simplified problem under the conditions that in the initial and final sections of the pipe, the fluid pressure is set in a complex form, as is done in the previous paragraph, which correspond to the case under consideration, i.e.

$$\begin{aligned} p &= \sum_{n=1}^N p_{n0} \exp(in\omega t) \text{ at } x = 0, \\ p &= \sum_{n=1}^N p_{nL} \exp(in\omega t) \text{ at } x = L. \end{aligned} \quad (10)$$

Here: p_{n0} and p_{nL} – vibration amplitudes; ω – circular vibration frequency; n – harmonic number.

The solution to the system of equations (9) is sought in the form

$$V_x(x, r, t) = \bar{V}_x(r) e^{in\omega t},$$

$$p_x(x, t) = \bar{p}(x) e^{in\omega t}.$$

Then the system of equations takes the following form:

$$\frac{\partial^2 \tilde{V}_x}{\partial r^2} + \frac{1}{r} \frac{\partial \tilde{V}_x}{\partial r} - \frac{in\omega}{v} \tilde{V}_x = \frac{1}{\rho v} \frac{\partial \tilde{p}}{\partial x}, \quad (11)$$

$$\frac{\partial p}{\partial r} = 0, \quad \frac{\partial \tilde{V}_x}{\partial x} + \frac{\partial \tilde{V}_r}{\partial r} + \frac{\tilde{V}_r}{r} = 0, \quad (12)$$

$$in\omega \frac{1}{a} \bar{p} = -\frac{\partial \bar{V}_x}{\partial x}, \quad (13)$$

where $a = \frac{\bar{E}h}{2R}$,

The solution of the system of equations (11), (12) and (13) taking into account the boundary conditions (4) is written in the form

$$\bar{V}(x, r) = \frac{1}{\rho(in\omega)} \left(-\frac{\partial \bar{p}}{\partial x} \right) \left(1 - \frac{I_0 \left(\sqrt{\frac{in\omega}{v}} r \right)}{I_0 \left(\sqrt{\frac{in\omega}{v}} R \right)} \right). \quad (14)$$

Multiply both sides of formula (14) by $\frac{2r}{R^2}$ and integrate from 0 to R, we get

$$\bar{V}(x) = \frac{1}{in\omega\rho} \left(-\frac{\partial\bar{p}}{\partial x} \right) \left(1 - \frac{2J_1(i^{3/2}\alpha_n)}{i^{3/2}\alpha_n J_0(i^{3/2}\alpha_n)} \right). \tag{15}$$

where $\alpha_n^2 = \frac{n\omega}{\nu} R^2$

we denote

$$z = \left[\frac{1}{in\omega\rho} \left(1 - \frac{2J_1(i^{3/2}\alpha_n)}{i^{3/2}\alpha_n J_0(i^{3/2}\alpha_n)} \right) \right]^{-1}. \tag{16}$$

then $\frac{\partial\bar{p}}{\partial x} = -z\bar{V}_x(x)$. (17)

Differentiating (17) with respect to x and substituting its value from equation (13) into place $\frac{\partial\bar{V}_x(x)}{\partial x}$, we obtain the equations for determining the pressure

$$\frac{\partial^2\bar{p}}{\partial x^2} - \frac{in\omega}{a} z\bar{p} = 0. \tag{18}$$

for this equation, the boundary conditions are

$$\begin{aligned} \bar{p} &= \sum_{n=1}^N \bar{p}_{n0} \text{ at } x = 0, \\ \bar{p} &= \sum_{n=1}^N \bar{p}_{nL} \text{ at } x = L. \end{aligned} \tag{19}$$

The solution to equation (18), taking into account the boundary conditions (19), has the following form:

$$\bar{p}(x) = \sum_{n=1}^N \left[\bar{p}_{n0} \frac{sh\sqrt{\frac{in\omega}{a}} zL \left(1 - \frac{x}{L}\right)}{sh\sqrt{\frac{in\omega}{a}} zL} + \bar{p}_{nL} \frac{sh\sqrt{\frac{in\omega}{a}} zL \frac{x}{L}}{sh\sqrt{\frac{in\omega}{a}} L} \right], \tag{20}$$

$$\bar{V}_x(x) = \sum_{n=1}^N \left[p_{n0} \frac{ch\sqrt{\frac{in\omega}{a}} zL \left(1 - \frac{x}{L}\right)}{sh\sqrt{\frac{in\omega}{a}} zL} - \bar{p}_{nL} \frac{sh\sqrt{\frac{in\omega}{a}} zL \frac{x}{L}}{sh\sqrt{\frac{in\omega}{a}} zL} \right] \sqrt{\frac{in\omega}{a} z}. \tag{21}$$

Analysis of the results obtained and their discussion

From the obtained formulas (20) and (21), it can be seen that the velocity and pressure significantly depend on the complex parameter $\sqrt{\frac{in\omega}{a}} zL$.

Therefore, we denote the complex parameter by $\bar{\chi} + \bar{\beta}i$, i.e.

$$\sqrt{\frac{i\omega}{a}} zL = \bar{\chi} + \bar{\beta}i \tag{22}$$

For simplicity of the problem, we take $n = 1$. Then

$$\sqrt{\frac{i\omega}{a}} zL = \bar{\chi} + \bar{\beta}i \tag{23}$$

$$\text{here } a = \frac{\bar{E}h}{2R}, \quad \bar{E} = \frac{E}{(1+v_1^2)}, \quad \alpha = \sqrt{\frac{\omega}{v}}R$$

$$z = \left[\frac{1}{i\omega\rho} \left(1 - \frac{2J_1(i^{3/2}\alpha)}{i^{3/2}\alpha J_0(i^{3/2}\alpha)} \right) \right]^{-1}$$

separating the real and imaginary parts of expression (23), we obtain

$$\bar{\chi} = \pm\omega\sqrt{\frac{\rho}{a}}L\sqrt{M_2^2 + N_2^2} \sin \frac{\varphi}{2},$$

$$\bar{\beta} = \pm\omega\sqrt{\frac{\rho}{a}}L\sqrt{M_2^2 + N_2^2} \cos \frac{\varphi}{2},$$

$$\text{where } \varphi = \text{arctg} \frac{N_2}{M_2}.$$

$$\text{here } J_0(i^{3/2}\alpha) = \text{ber}_0\alpha + \text{ibe}_0\alpha, \quad J_1(i^{3/2}\alpha) = \text{ber}_1\alpha + \text{ibe}_1\alpha,$$

$$\frac{J_1(i^{3/2}\alpha)}{J_0(i^{3/2}\alpha)} = M_1 + N_1i,$$

$$M_1 = \frac{\text{ber}_1\alpha\text{ber}_0\alpha + \text{bei}_1\alpha\text{bei}_0\alpha}{\text{ber}_0^2\alpha + \text{bei}_0^2\alpha}, \quad N_1 = \frac{-\text{ber}_1\alpha\text{bei}_0\alpha + \text{ber}_0\alpha\text{bei}_1\alpha}{\text{ber}_0^2\alpha + \text{bei}_0^2\alpha},$$

$$\frac{2}{i^{3/2}\alpha} = \frac{2}{i\sqrt{i}\alpha} = -\frac{\sqrt{2}}{\alpha}(1+i)$$

$$\begin{aligned}
 &-\frac{\sqrt{2}}{\alpha}(1+i)(M_1+N_1i) = -\frac{\sqrt{2}}{\alpha}(M_1-N_1) - \frac{\sqrt{2}}{\alpha}(M_1+N_1)i \\
 &1 + \frac{\sqrt{2}}{\alpha}(M_1-N_1) + \frac{\sqrt{2}}{\alpha}(M_1+N_1)i, \\
 z = i\omega\rho &\left[1 - \frac{2J_1(i^{3/2}\alpha)}{i^{3/2}\alpha J_0(i^{3/2}\alpha)}\right]^{-1} = i\omega\rho \left[\frac{1}{\left(1 + \frac{\sqrt{2}}{\alpha}(M_1-N_1)\right) + \left(\frac{\sqrt{2}}{\alpha}(M_1+N_1)i\right)} \right] = \\
 &= i\omega\rho \left[\frac{1 + \frac{\sqrt{2}}{\alpha}(M_1-N_1) - \frac{\sqrt{2}}{\alpha}(M_1+N_1)i}{\left(1 + \frac{\sqrt{2}}{\alpha}(M_1-N_1)\right)^2 + \left(\frac{\sqrt{2}}{\alpha}(M_1+N_1)\right)^2} \right], \\
 M_2 &= \frac{1 + \frac{\sqrt{2}}{\alpha}(M_1-N_1)}{\left(1 + \frac{\sqrt{2}}{\alpha}(M_1-N_1)\right)^2 + \left(\frac{\sqrt{2}}{\alpha}(M_1+N_1)\right)^2}, \\
 N_2 &= \frac{-\frac{\sqrt{2}}{\alpha}(M_1+N_1)}{\left(1 + \frac{\sqrt{2}}{\alpha}(M_1-N_1)\right)^2 + \left(\frac{\sqrt{2}}{\alpha}(M_1+N_1)\right)^2}.
 \end{aligned}$$

then

$$z = i\omega\rho(M_2 + N_2i),$$

$$\sqrt{\frac{i\omega z}{a}}L = i\omega\sqrt{\frac{\rho}{a}}L(M_2 + N_2i)^{\frac{1}{2}} = \pm i\omega\sqrt{\frac{\rho}{a}}L\sqrt{M_2^2 + N_2^2} \left(\cos\frac{\varphi}{2} + i\sin\frac{\varphi}{2} \right).$$

here χ -coefficient characterizing the damping of oscillations, $\frac{1}{\beta}$ -dimensionless coefficient is the velocity of propagation of the pulse wave. If we denote the speed of propagation of the pulse wave through c , then $\frac{c}{c_\infty} = \frac{\omega L}{c_\infty \beta}$, where $c_\infty = \sqrt{\frac{Eh}{2\rho R}}$ - Moens-Korteweg formula; E - elastic modulus; h - wall thickness; ρ - fluid density; R - pipe radius; L - pipe length.

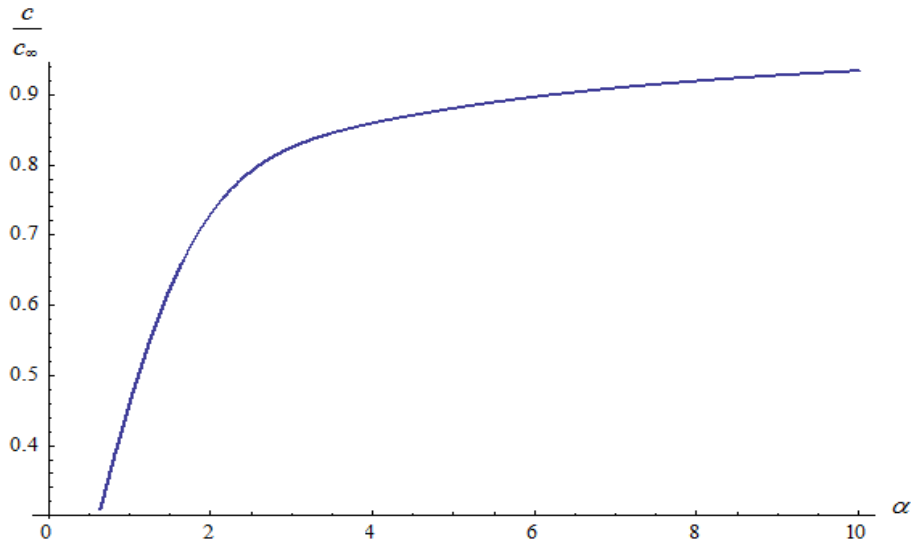


Figure: 1. Dependence of the dimensionless value of the phase velocity on the vibrational number α .

Based on the obtained formulas, we analyze the speed of propagation of the pulse wave and its attenuation depending on the vibrational Reynolds number.

In fig. 1.shows the dependence of the dimensionless value of the pulse pressure wave on the vibrational number α . It was found that the speed of propagation of the pressure pulse wave increases with an increase in the elastic modulus of the surrounding tissue and an increase in wavelength. Here, the pulse wave velocity is also compared with the Moens-Kortewegvelocity C_∞ , and revealed significant differences between them occur at lower values of the Womersley vibrational parameter, at large values of which significant differences are not observed.

In fig. 2.shows the dependence of the reciprocal of the damping, referred to the wavelength, on the vibrational number α .

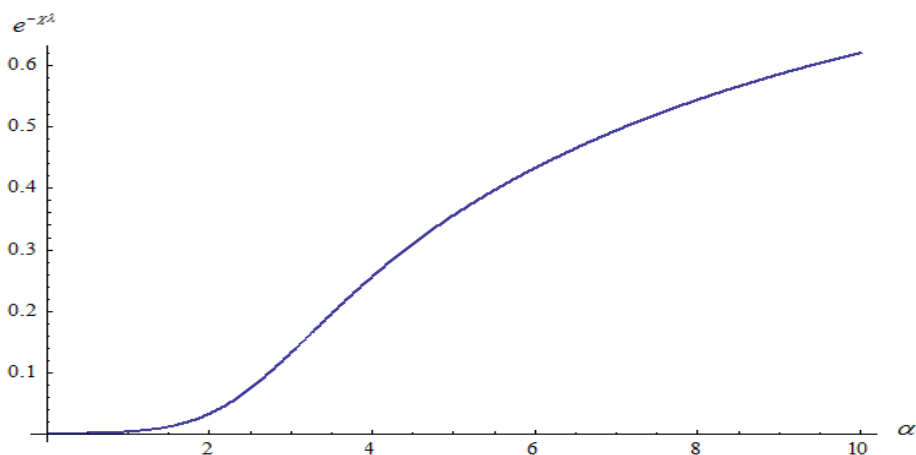


Figure: 2. Dependence of the reciprocal of the attenuation, referred to wavelength, from vibrational number α .

The results show that the damping is free at lower values of the Womersley vibrational parameter, practically equal to zero, and at its large values, it asymptotically approaches unity.

Conclusion

The dependence of the dimensionless value of the pressure pulse wave on the vibrational number was

investigated α . It was found that the speed of propagation of the pressure pulse wave increases with an increase in the elastic modulus of the surrounding tissue and an increase in wavelength. The speed of the pulse wave was compared with the speed of Moens-Korteweg C_∞ , and significant differences were revealed between them occurring at lower values of the Womersley oscillatory parameter, at large values of which significant differences are not observed. The dependence of the reciprocal damping per wavelength on the vibrational number α , is also investigated; it is shown that the damping is free at lower values of the Womersley vibrational parameter, practically equal to zero, and at large values of it, asymptotically approaches unity. The presented simplified model is suitable for determining the speed of propagation of a pulse wave and attenuation of pulses. However, it is not acceptable for determining the hydraulic resistance in an elastic pipe, since

in this case the impedance $\left(-\frac{\partial p}{\partial x}\right)/Q$

does not depend on the wall elasticity coefficient. In order to determine the hydraulic resistance in an elastic pipe, it is necessary to solve the problem in a two-dimensional formulation, that is, taking into account the orthotropy of the wall deformation, using the linearized Navier-Stokes equations for the flow of a viscous fluid.

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