Integration of mathematical methods

Saipnazarov Shaylovbek Aktamovich*;
Khodjabaeva Dilbar;**
Usarov Jurabek Abdunazirovich***
*Associate Professor, Candidate of Pedagogical Sciences, UZBEKISTAN
Email id: shaylovbek.s.a@gmail.com; https://orcid.org/0000-0003-4280-6304
** Senior Lecturer, Tashkent University of Economics, UZBEKISTAN Email id: dxodjabaeva@gmail.com; https://orcid.org/0000-0003-0584-0642
*** Tashkent University of Economics, UZBEKISTAN Email id: jurabek0117@gmail.com; https://orcid.org/0000-0002-5351-4211

ABSTRACT
This article discusses solutions to algebraic problems with the trigonometry method. It also shows the problems of mathematical analysis with the method of geometry and solving algebraic problems with the method of mathematical analysis using the derivative.

Keywords
Trigonometric substitution, root, interval, systems of equations, variable, mathematical education, expression.
Article Received: 18 October 2020, Revised: 3 November 2020, Accepted: 24 December 2020

Introduction

In modern knowledge of the material world, one of the leading places rightfully belongs to mathematics, which has become the most important means of integrating sciences and scientific knowledge. The enormous synthetic potential of mathematics lies in its abstraction, allowing to break away not only from the phenomena and processes of reality, but also from their own positions and conclusions. This allows her and with her help other sciences to build new objects of knowledge with the disclosure of their supposed essence.

Integrative processes in higher mathematical education developed, in accordance with the laws of dialectics: the unity and struggle of opposites, denial and the transition of quantitative changes to qualitative ones. Integration, as a phenomenon, can be a mechanical sum of combined elements, in this case it characterizes the beginning of integration processes. Their further development leads to the fact that integration acts as on organic unity of the combined elements-each part is implanted into the whole. In accordance with these principles, integrated training courses, including mathematical ones, are created.

1. Trigonometric method for solving algebraic problems

This method involves the use of equations, inequalities and their systems. Often, when solving algebraic problems, it is convenient to replace a variable with trigonometric function and thereby reduce an algebraic problem to a trigonometric one. Such substitutions, trigonometric substitutions, sometimes greatly simplify the solution.

The choice of this or that function in this case depends on the type of equation, system of equations or algebraic expression that needs to be simplified. For example, if it follows from the conditions that the admissible values of the variable X are determined by inequality $|x| \le 1$, then it is convenient to replace

$$x = \sin\alpha, \alpha \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right] \quad \text{or} \quad x = \cos\alpha, \alpha \in [0; \pi]$$

and which one choose depends on the specific problem.

In cases where a variable can take any values, replacements $x = \mathrm{tg}\alpha, \alpha \in \left(-\frac{\pi}{2};\frac{\pi}{2}\right)$, or $x = \mathrm{ctg}\alpha, \alpha \in (0;\pi)$.

Example 1.1. Solve the equation

$$\sqrt{1-x} = 2x^2 - 1 + 2x\sqrt{1-x^2}$$

Solution. In this equation, $|x| \le 1$. Let $x = \cos\alpha$, where $\alpha \in [0;\pi]$. Making the substitution and taking into account that $\sin\frac{\alpha}{2} \ge 0$, $\sin\alpha \ge 0$, we obtain the equation

$$\sin\frac{\alpha}{2} = \cos 2\alpha + \sin 2\alpha,$$

which is converted to the from

$$\sin\left(\frac{3\alpha}{4} + \frac{\pi}{8}\right)\cdot\cos\left(\frac{5\alpha}{4} + \frac{\pi}{8}\right) = 0,$$

From where

$$\alpha = -\frac{\pi}{6} + \frac{4\pi k}{3}, k \in Z, \text{ or } \alpha = \frac{3\pi}{10} + \frac{4\pi n}{5}, n \in Z$$

Of the obtained values of α in the interval $(0;\pi)$ there is only one

$$\alpha = \frac{3\pi}{10}, \text{ so } x = \cos\frac{3\pi}{10}.$$

The answer can be left in trigonometric form, but it can also be expressed in radicals, using formula $\sin 18^0 = \frac{\sqrt{5}-1}{4}$, which is derived from the obvious equality $\sin 36^0 = \cos 54^0$.

$$\sin 18^0 = \sin\frac{\pi}{10} = \frac{\sqrt{5}-1}{4}, \sin^2\frac{\pi}{10} = \frac{3-\sqrt{5}}{8}$$

$$\frac{1-\cos\frac{2\pi}{10}}{2} = \frac{3-\sqrt{5}}{8}, \cos\frac{2\pi}{10} = 1 - \frac{3-\sqrt{5}}{4} = \frac{1+\sqrt{5}}{4}$$

$$\sin\frac{2\pi}{10} = \cos\frac{3\pi}{10} = \sqrt{1-\left(\frac{1+\sqrt{5}}{4}\right)^2} = \frac{1}{4}\sqrt{10-2\sqrt{5}}$$

Example 1.2. How many roots on the segment [0;1] does the equation have $8x(1-2x^2)(8x^4 - 8x^2 + 1) = 1$

Solution. This in an equation of the seventh degree, so attempts to solve it by algebraic methods are factor are certainly hopeless. However, the presence of the factor $1-2x^2$ suggests to make *a* trigonometric substitution $x = \cos\alpha$, $\alpha \in (0;\frac{\pi}{2})$ (it's obvious that $x \ne 0$, $x \ne 1$).

Replacing x on the left side of this equation by $\cos\alpha$, we obtain

$$8\cos\alpha(1-2\cos^2\alpha)(8\cos^4\alpha - 8\cos^2\alpha + 1) = 8\cos\alpha(-\cos 2\alpha)(8\cos^2\alpha(\cos^2\alpha - 1) + 1) =$$

$$= -8\cos\alpha\cos 2\alpha(-2\sin^2 2\alpha + 1)$$

, from where $-8\cos\alpha\cos2\alpha\cos4\alpha=1$ (*) Now multiply both of (*) by $\sin\alpha \quad (\alpha\neq 0)$:

$$-8\cos\alpha\cos2\alpha\cos4\alpha=\sin\alpha,$$

OR

$$-\sin8\alpha=\sin\alpha,$$
$$-\sin8\alpha+\sin\alpha=0,$$
$$2\sin\frac{9}{2}\alpha\cos\frac{7}{2}\alpha=0,$$

from where $\alpha=\frac{2}{9}\pi k,\ k\in z,$ or $\alpha=\frac{\pi}{7}+\frac{\pi n}{7},\ n\in z.$

From the first series, the interval $\left(0;\frac{\pi}{2}\right)$ contains two values: in the second series, there are also two such values: $\alpha_3=\frac{\pi}{7},\ \alpha_4=\frac{3\pi}{7}$.

Answer: The original equation has exactly four roots on the interval [0;1]

Example 1.3. Among all the solutions (x,y,z,v) of the system

$$\begin{cases} x^2+y^2=4, \\ z^2+v^2=9, \\ xv+yz\geq 6 \end{cases}$$

Find those for which the expression x+z takes the largest value.

Solution. Suppose that $x=2\sin\alpha$, $y=2\cos\alpha$, $z=2\sin\beta$, $v=2\cos\beta$. We obtain the following inequality

$$2\sin\alpha\cdot3\cos\beta+2\cos\alpha\cdot3\sin\beta\geq 6,$$
$$\sin\alpha\cdot\cos\beta+\cos\alpha\cdot\sin\beta\geq1\Rightarrow\sin(\alpha+\beta)=1,\ \text{or}$$
$$\alpha+\beta=\frac{\pi}{2}+2\pi n,\ n\in Z$$
$$x+z=2\sin\alpha+3\sin\beta=2\sin\alpha+3\sin\left(\frac{\pi}{2}+2\pi n-\alpha\right)=2\sin\alpha+3\cos\alpha=\sqrt{2^2+3^2}=\sqrt{13},$$
$$x=\frac{4}{\sqrt{13}},\ z=\frac{9}{\sqrt{13}}$$

Example 1.4. Find all triples of real numbers ($x,y,z,$) that satisfy the system

$$\begin{cases} 2x+x^2y=y, \\ 2y+y^2z=z, \qquad (**) \\ 2z+z^2x=x \end{cases}$$

Solution. It is easy to verify that the system

$$y = \frac{2x}{1-x^2}$$
$$z = \frac{2y}{1-y^2}$$
$$x = \frac{2z}{1-z^2}$$

is equivalent to the original.

We know the formula $\mathrm{tg}2\alpha = \frac{2tg\alpha}{1-tg^2\alpha}$.

This suggest doing the trigonometric substitution $x = \mathrm{tg}\alpha$, where $\alpha \in \left(-\frac{\pi}{2};\frac{\pi}{2}\right)$, $\alpha \neq -\frac{\pi}{4}$, $\alpha \neq \frac{\pi}{4}$. From system (**) we obtain $x = \mathrm{tg}\alpha$, $y = \mathrm{tg}2\alpha$, $z = \mathrm{tg}4\alpha$, $x = \mathrm{tg}8\alpha$. So, $\mathrm{tg}8\alpha = \mathrm{tg}\alpha$, $8\alpha = \alpha + \pi n$, $\alpha = \frac{\pi n}{7}$, $n \in Z$

2.Geometric method for solving problems of mathematical analysis

Example 2.1. Find the smallest value of the

$$f(x) = \sqrt{5x^2+20} + \sqrt{5x^2-32x+64} + \sqrt{5x^2-40x+100} + \sqrt{5x^2-8x+16}$$

Solution. Putting $\vec{a} = (5x,10)$, $\vec{b} = (16-5x,8)$, $\vec{c} = (20-5x,10)$, $\vec{d} = (5x-4,8)$,

we get

$$\sqrt{5}f(x) = |\vec{a}| + |\vec{b}| + |\vec{c}| + |\vec{d}| \geq |\vec{a}+\vec{c}| + |\vec{b}+\vec{d}| = |(20;20)| + (12;16) =$$
$$= \sqrt{400+400} + \sqrt{144+256} = 20\sqrt{2} + 20$$

in this case, equality is attained for $\vec{a}\, II\, \vec{c}$ and $\vec{b}\, II\, \vec{d}$, i.e. at

$$\frac{5x}{10} = \frac{20-5x}{10};\ \frac{16-5x}{8} = \frac{5x-4}{8}, \text{ or } x=2.$$

So the smallest value of *f*(*x*) is $f(2) = 4\sqrt{5}(1+\sqrt{2})$ of the function

Example 2.2. Find the smallest value of the function

$$f(x,y) = \sqrt{x^2+(9-y)^2} + \sqrt{y^2+(12-x)^2}$$

where *x* and *y* are real numbers

Solution. Consider the points *B*(0,9), *C*(12,0), *M*(*x*,*y*).

Then $BM = \sqrt{x^2+(9-y)^2}$; $CM = \sqrt{(12-x)^2+y^2}$

i.e. given function *f*(*x*,*y*) equals *f*(*x*,*y*)=*BM*+*MC*.

It follows from the triangle inequality that $f(x,y) = BM + MC \geq \sqrt{9^2+12^2} = 15$

3.Method of mathematical analysis for solving algebraic problems.

Example 3.1. Factor the expression $xy(x-y) + yz(y-z) + xz(z-x)$

Solution. Assuming *x* to be *a* variable, consider the function

$$f^{/}(x) = 2xy - y^2 + z^2 - 2xz = (y-z)(2x-y-z)$$

as

$$(x^2-(y+z)\cdot x)^{/} = 2xy - y - z, ((y-z)(x^2-(y+z)x))^{/} = (y-z)(2x-y-z),$$

then from this we conclude that

$$f^{/}(x) = ((y-z)(x^2-(y+z)x)^{/}$$

then

$$f(x) = (y-z)(x^2-(y+z)x)+C,$$

where C does not depend on x, but depends, generally speaking, on y and z since the last equality is true for any x, then, from example, if x=0 in it and taking into account that $f(0)=yz(y-z)$, we find C= $yz(y-z)$.

In this way

$$f(x) = (y-z)(x^2-(y+z)x)+yz(y-z) =$$

$$= (y-z)(x^2-(y+z)x+yz) = (y-z)(x(x-y)-z(x-y)) =$$

$$= (x-y)(y-z)(x-z)$$

So,

$$xy(x-y)+yz(y-z)+xz(z-x) = (x-y)(y-z)(x-z).$$

Example 3.2. Solve the equation $x^3-8x^2+13x-6=0$.

Solution. Consider the polynomial

$$f(x) = x^3-8x^2+13x-6,$$

whose derivative is

$$f^{/}(x) = 3x^2-16x+13.$$

Find the greatest common divisor of the polynomials $f(x)$ and $f^{/}(x)$.

We have

$$x^3-8x^2+13x-6 = (3x^2-16x+13)\left(\frac{1}{3}x-\frac{8}{9}\right)-\frac{50}{9}(x-1),$$

$$3x^2-16x+13 =)(x-1)(3x-13)$$

Thus, the greatest common divisor of $f(x)$ and $f^{/}(x)$ is x-1.

Since x=1 is a simple root of the greatest common divisor, the number x=1 will be a double root of this equation, and, therefore, the polynomial $f(x)$ is divisible by $(x-1)^2$, we find that $f(x)=(x-1)^2(x-6)$. Therefore, the roots of the original equation are the numbers $x_1=x_2=1$ and x=6 and only they.

Example 3.2. Solve the system of equation

$$\begin{cases} x^2y+2xy^2+y^3=9, \\ x^3y+y^4=7. \end{cases} \quad (1)$$

Solution. Let us rewrite this system as

$$\begin{cases} y(x+y)^2=9, \\ y(x^3-y^3)=7. \end{cases} \quad (2)$$

Is divisible by It follows from the first equation of this system that its solutions can be such pairs of numbers must satisfy the inequality x>y>0, which follows from the second equation of the system (2). Let $\mathrm{t}=\sqrt{\mathrm{y}}$; then from the first equation of the system we find that $\mathrm{x}=\frac{3}{\mathrm{y}}=t^2.$

Substituting in the second equation of the system $\frac{3}{\mathrm{y}}=t^2$ together x and t^2 instead of y, we get

$$t^2\left(\left(\frac{3}{t}-t^2\right)^3-t^6\right)=7, \text{or}$$

$$\left(3-t^3\right)^3 - t^9 - 7t = 0 \qquad (3)$$

As

$$\left(\left(3-t^3\right)^3 - t^9 - 7t\right)' = -9\left(3-t^2\right)^2 - 9t^8 - 7 = -\left(9\left(3-t^2\right)^2 \cdot t^2 + 9t^8 + 7\right) < 0,$$

then equation (3) has at most one root. It is easy to see that the number t=1 is a root. From here we find that the solution to this system can only be a pair of numbers x=2 and y=1. By checking, we make sure that this pair of numbers really is a solution to system (1).

Conclusion

Education differs from teaching in that it involves mastering the methods of scientific activity, and not only the content that is set forth in university textbooks and fixed in programs. The role of education is also to achieve an understanding of the connections and coherence between diverse fields of knowledge and experience.

This approach to the study of mathematics, through the study of its constituent methods of cognition, allows you to influence not only mathematical, but also the general, intellectual and cultural development of students.

The integration of the subject of cognition is accompanied by the use of an increasing complex of means and method of theoretical and empirical development of the object.

The solution of complex problem is possible with the help of any one theoretical method and technical means of research, this requires the interconnection of many such methods and means, an over deeper interconnection of theory and empiricism.

The integrative processes intensively occurring in the named components of scientific activity lead to the synthesis of its results. In general, a synthesis, the integration of forms of scientific disciplines turn out to be interconnected theories hypotheses and other logical forms, here the differentiation of the results of cognition is in close connection with integration.

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IDENTICAL TRANSFORMATIONS IN PROBLEM SOLVING. ACADEMICIA: An International Multidisciplinary Research Journal https://saarj.com 1. ISSN: 2249-7137 Vol. 10, Issue 10, October 2020