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Explain Point and Line Positioning Materials Using the Ethnomathematical Approach to Enhance Students' Geometric Thinking Skills

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ABSTRACT

Quality learning materials are a necessity to improve the quality of mathematics learning. Quality materials can be obtained through research and development. This study developed learning materials based on the van-Hiele learning phases, integrated with the ethnomathematics approach. In this case, the ethnomathematics used were in the form of a checkerboard game. The development model was based on the Plomp model. The results of the analysis showed that the developed material is valid and practical. The product trials results showed that the application of learning using the van-Hiele phase integrated with the checkerboard game could significantly improve students' understanding of the point and line position material. The lecturers expected the lecturers to provide quality materials and integrate them with the ethnomathematics approach based on the findings.

Keywords

ethnomathematics, van-Hiele learning, point, line positions

Introduction

As stated in the National Standards for Higher Education in Indonesia, the characteristics of the learning process cover holistic, interactive, contextual, integrative, scientific, effective, collaborative, thematic, and student centred. Holistic means the learning process encourages forming a comprehensive and broad mindset by internalizing both local and national wisdom. Indonesia is a country with diverse ethnicities, cultures, and religions, and its population is more than 270,000,000 people (Hendriyanto, Kusmayadi, & Fitriana, 2021). Its geographic location stretches across a cluster of small islands with diverse ethnic and cultural characteristics (Prahmana & Ambrosio, 2020; Utami, Sayuti, & Jailani, 2019). The diverse ethnicity and cultures result in varied cultural products in dance, language, traditional houses, games, and other types of community activities (Mardia, Turmudi, & Nurjanah, 2020; Maryati & Prahmana, 2019; Pramudita & Rosnawati, 2019; Roza, Siregar, & Solfitri, 2020).

Mathematics is one of the elements that form the basis of modern science and technology (Muhtadi, Sukirwan, Warsito, & Prahmana, 2017). Mathematics is a universal science used in various aspects of life (Suharta, Sudiarta, & Astawa, 2017). Mathematics is a scientific field that is closely related to life activities, where every activity is always a mathematical activity (Hartono & Saputro, 2019; Nurhasanah, Kusumah, & Sabandar, 2017; Prahmana, Zulkardi, & Hartono, 2012). Therefore, learning mathematics should have relevant value for constructing mathematical concepts in real life (Nugraha, Maulana, & Mutiasih, 2020). Learning mathematics that is integrated with community culture increases students' ability to explore mathematical concepts (Brandt & Chernoff, 2015; Rosa & Orey, 2017). Moreover, Supiyati, Hanum, & Jailani (2019) revealed that culture is a special way for individuals to adapt to their environment, while mathematics is realized due to individuals' acts. Munthahana & Budiarto (2020) believed that inserting culture in learning will motivate students to recognize their culture and be proud of their cultural identity.

In general, students have always considered mathematics to be difficult (Nuari, Prahmana, & Fatmawati, 2019). Thus, the students' achievement in international assessments is one of the main concerns of the Indonesian Government and researchers in mathematics (Hidayah & Forgasz, 2020). The students' low learning achievement in mathematics is not only a concern of Indonesia but has become a global concern for years (Sunzuma & Maharaj, 2020). It may become the reason for educators' inconsistency who do not take advantage of the local environment in learning (Abdullah, 2017). Besides, Kariman, Harisman, Sovia, & Prahmana (2019) state that the factors that become obstacles in the development of education are gaps between the administration and the education system and the weak readiness of educators to teach.

University students are expected to use mathematics well both in earning and daily life (Simamora, Saragih, & Hasratuddin, 2018). Next, Yanti & Haji (2019) state that human activities expressing their ideas on cultural objects often have mathematical values. Community culture can be contributed as supporting mathematics learning for students. There are many types of traditional games that require mathematical concepts in geometry, algebra, statistics, etc. Games are fun activities for everyone, including students. Indonesia has many traditional games, but they are rarely played now because of the domination of modern games (Dele-Ajayi, Strachan, Pickard, & Sanderson, 2019; Nasrullah & Zulkardi, 2011). Some previous studies on ethnomathematics concern traditional games Asrial, Syahrial, Maison, Kurniawan, & Perdana, (2020); Febriyanti, Prasetya, & Irawan (2018); Fendrik, Marsigit, & Wangid (2020); Fouze & Amit (2018); Risdiyanti, Prahmana, & Shahrill (2019); Turmudi, Susanti, Rosikhoh, & Marhayati (2021).

Literature Review

Van-Hiele Learning

Vojkuvkova (2012) says that Pierre van Hiele and his wife Dina van-Hiele Geldof are researchers and teachers in the Netherlands. They have personal experiences related to their students' difficulties in studying geometry. Therefore, they examined this issue in detail. Van-Hiele's theory comes from their research at Utrecht University in 1957, which was conducted in the Soviet Union in the 1960s. It resulted in the creation of a successful new geometry curriculum designed in the USSR. Crowley and Usiskin explained that in van-Hiele's theory, geometric thinking is categorized into three main components of a) the level of geometric thinking, b) characteristics, and c) the learning phase (Meng & Idris, 2012).

Levels of Geometric Thinking

Van-Hiele categorized geometric thinking levels sequentially into five: covering visualization, analysis, informal deduction, deduction, and rigor (Baiduri, Ismail, & Sulfiyah, 2020; Nisawa, 2018). The characteristics of each geometric thinking level based on van Hiele's theory are presented in Table 1.

Table 1. Characteristics of van-Hiele's Geometric Thinking Skills

Level	Characteristics
Level 1 (visualization)	Students can recognize and visualize geometric shapes based on characteristics of visual appearance.
Level 2 (analysis)	Students begin to identify the properties and elements of a geometric shapes or idea
Level 3 (informal deduction)	Students can find and prove relationships between geometric shapes' properties and elements, both in one image and some images.
Level 4 (deduction)	Students can understand the interaction between necessary and sufficient requirements and have the opportunity to find solutions to existing issues.
Level 5 (rigor)	Students can use the data obtained at the previous level in different axioms to find solutions to the issues given.

Source: adopted from Karakus & Paker, 2015, p. 339 cited by (Armah & Kissi, 2019; Salifu, 2018)

Characteristics of van-Hiele's Geometric Thinking

Van-Hiele's learning theory's characteristics cover sequential, intrinsic and extrinsic, linguistics, mismatch, and advancement (Rouadi, 2014). The explanation of each characteristic can be seen below.

1. Sequential: Students show progress sequentially from one level to a higher level, in which students will not reach the level of deduction without achieving the ability at the previous level - informal deduction.
2. Intrinsic and extrinsic: Ideas understood at level 2, for example, should be the subject of further study at level 3.
3. Linguistics: Each level has its language, vocabulary, symbols, and relationships. Van-Hiele considers, in thinking, the role of language is significant. Without language, it is impossible to think. The development of knowledge will not occur without language (Alex & Mammen, 2016).
4. Mismatch: If students at certain levels are taught specific skills above their cognitive level and do not follow the order (the students' ability does not match the student's level), they cannot acquire knowledge.
5. Advancement: Students can learn and acquire skills above their level without knowing; for example, they can learn to count fractions without knowing that it is a fraction and what the fraction represents.

Van-Hiele's Learning Phases

In learning geometry, van-Hiele proposed some phases of learning to help students improve to a higher level. The phases are information, guided orientation, explication, free orientation, integration (Mdyunus & Hock, 2019). Armah & Kissi (2019) describe each phase as follows.

1. Information, educators introduce students to the direction of the study. Educators assess the students' interpretations/reasoning.
2. Guided orientation, students participate actively in exploring objects to obtain principal relationships of the formed understanding. Educators only act as facilitators.
3. Explication introduces terminology related to geometry and requires students to use it in the presentation and completion of assignments.
4. Students are given more complex tasks in free orientation and can find their way to complete each task. Thus, they become more proficient with the knowledge they have before.
5. Integration, the material that has been studied is summarized by the students and then create an overview of existing concepts.

Ethnomathematics Approach

Rosa & Orey (2016) explain that ethnomathematics is composed of three Greek words, namely *ethno*, which means natural or socio-cultural groups, and *mathema*, which means explaining and learning. Moreover, *tics* means method, art, and technique. Thus, ethnomathematics is interpreted as a program related to the motives in which members of a specific culture *(ethno)* develop, throughout history, measure, calculate, infer, compare, and classify techniques and ideas *(mathema)* that enable them to model the environment in natural and social contexts to explain and understand a phenomenon *(tics)*.

Patrianto & Rahiem (2019) quote Powel's opinion, which stated that "the mathematics which is practiced among identifiable cultural groups such as national-tribe societies, labour groups, children of certain age brackets and professional classes." Powel defines ethnomathematics as mathematics that is applied among cultural groups that can be identified, such as labour groups, professional classes, children, and ethnic groups. Furthermore, some Indonesian researchers such as Haryanto,Nusantara, Subanji, & Rahardjo (2017); Muhtadi et al., (2017); Risdiyanti & Prahmana (2020) argue that ethnomathematics is the application of mathematics in culture. Long & Chik (2020) state that ethnomathematics studies aim to determine culture and mathematics related to everyday life. Besides, ethnomathematics is culture-oriented learning and aims to explore mathematical concepts in community socio-cultural activities (Rosa & Clark, 2011; Tereshkina, Merlina, Kartashova, Dyachkovskaya, & Pyryrco, 2015).

Therefore, it can be said that ethnomathematics is an approach to learn mathematics by applying cultural values that can be identified from various cultural groups such as community activities, religion, work, the community of children at a certain age, indigenous peoples, etc. (Supriadi, 2019) reveals that the mathematical concept based on a cultural perspective allows students to reflect and appreciate their own culture and the culture and traditions of others.

Traditional Games of Dam-Dam

Susanti, Sholikin, Marhayati, & Turmudi (2020) explain that "dam-dams" is a traditional game in Indonesia that two people play to practice thinking skills and organize strategies. The procedure to play this game is similar to chess, but it uses much simpler equipment and method. This game uses 32 pieces divided into 16 gravels and 16 marbles or other objects with the same amount to show its identity.

Source: https://specialpengetahuan.blogspot.com/2015/02/permainan-tradisional-dam-daman-bas.html

Figure 1. Traditional Game "dam-dam"

Rules in the traditional game "dam-dam":

1. Two players play the game.
2. Each player arranges troops (16 pieces) on the game square before the game starts.
3. The players make a "suit" to determine who has the right to run the piece first.
4. Players take turns running the pieces while thinking of the strategy to eat the opponent's pieces.
5. If a player forgets not to eat the opponent's pieces when given the bait, then the opponent must say "dam" so that the opponent can take three pieces as he wishes (using strategies to take the potential pieces so that he/she can eat more pieces later on).
6. If one of the pieces can enter the opponent's triangle arena and go around three times, then the pieces can become a king to walk, jump without limitation on the number of squares passed and eat the opponent's pieces as desired.
7. Pieces can become king automatically if there is only one left.
8. The player wins the game if he/she can eat all the opponent's pieces.

Based on Figure 1, some geometry concepts can be identified, including points, lines, triangles, squares, rectangles.

Point and Line Position Materials in Dam-Dam Game

The concept of analytical geometry in the game "dam-dam" cannot be found directly. It is because analytical geometry is a combination of algebra and geometry, which relates to analytical thinking. Thus, to find the concept of analytical geometry in the game "dam-dam," the game has to be visualized in a coordinate system, as shown in Figure 2.

Figure 2. The visualization sample of the traditional game "dam-dam" in an orthogonal coordinate system.

Cartesian Coordinate System

First, to visualize checkerboard plots in a coordinate system, one must know the coordinate system and its elements. The Cartesian coordinate system is also called a 2-dimensional orthogonal coordinate system which is generally determined by two perpendicular axes (*X*-axis and the *Y*-axis) in which both of them are in one plane (*XY*-plane). Perpendicular to each other, the two axes divide the plane into four parts. To describe a certain point in a 2-dimensional coordinate system is by writing down the x (abscissa) value followed by y (ordinate) value with the format of (x, y). The intersection between the *X*-axis and the *Y*-axis is called the point $O\ (0,0)$ or the origin.

The essential thing in a coordinate system is the *X*-axis and the *Y*-axis perpendicular to each other. The checkerboard has many perpendicular lines that can describe the orthogonal coordinate system, for example, take two intersecting lines on the checkerboard as shown in Figure 3.

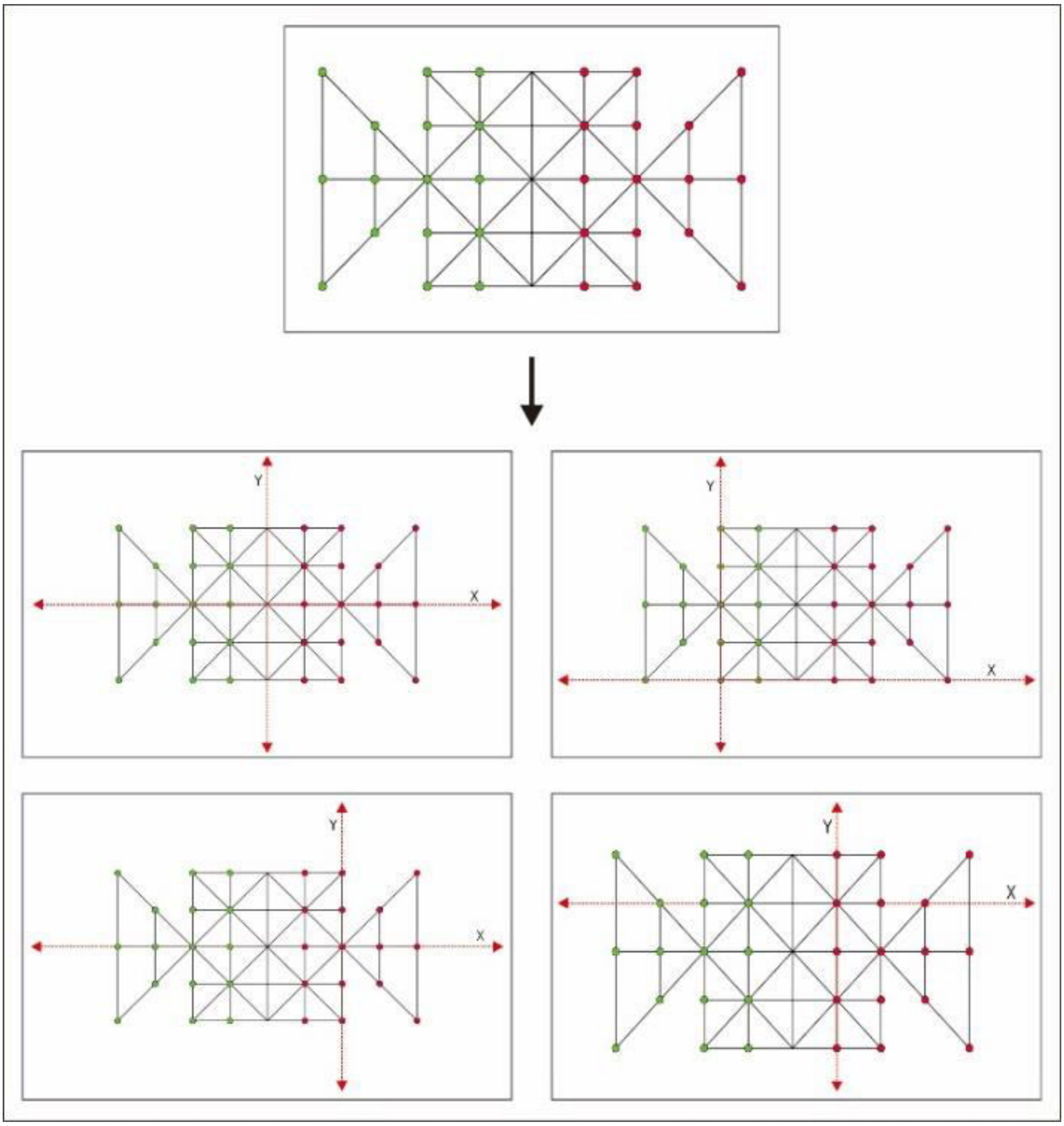

Figure 2. The visualization sample of the traditional game "dam-dam" in an orthogonal coordinate system

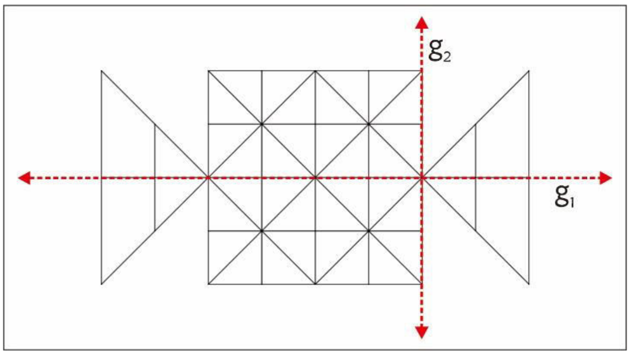

Figure 3. Sample of two intersecting lines

Theorem 1

The *X*-axis and *Y*-axis are perpendicular to each other

Evidence:

For example: $g_1(X - axis): y = 0$ and $g_2(Y - axis): x = 0$

$g_1: y = 0 \rightarrow g_1: y = r\sin\varphi = 0$ (1)

$g_2: x = 0 \rightarrow g_2: x = r\cos\varphi = 0$ (2)

Rotation g_1 and g_2 is counterclockwise with an angle of γ

$g_1: r\sin(\varphi - \gamma) = 0$ (3)

$g_1: r(\sin\varphi\cos\gamma - \cos\varphi\sin\gamma) = 0$ (4)

$g_1: r\sin\varphi\cos\gamma - r\cos\varphi\sin\gamma = 0$

Substitution (1) and (2)

$$g_1: y\cos\gamma - x\sin\gamma = 0 \quad (5)$$

$$mg_1 = \frac{\sin\gamma}{\cos\gamma} \quad (6)$$

$$g_2: r\cos(\varphi - \gamma) = 0 \quad (7)$$

$$g_2: r(\sin\varphi\sin\gamma + \cos\varphi\cos\gamma) = 0 \quad (8)$$

$$g_2: r\sin\varphi\sin\gamma + r\cos\varphi\cos\gamma = 0$$

Substitution (1) and (2)

$$g_2: y\sin\gamma + x\cos\gamma = 0 \quad (9)$$

$$mg_2 = -\frac{\cos\gamma}{\sin\gamma} \quad (10)$$

$$mg_1 \cdot mg_2 = \frac{\sin\gamma}{\cos\gamma} \cdot -\frac{\cos\gamma}{\sin\gamma} = -1 \quad (11)$$

Because the product of the two gradients $(m) = -1$, then g_1 and g_2 are perpendicular to each other after rotation. However, the angle between g_1 and g_2 is maintained during rotation, surely g_1 and g_2 are perpendicular. It means that $x = 0$ and $y = 0$ are perpendicular to each other.

Point Position

The position of a point in the Cartesian coordinate system is determined by the positive or negative of the *x* and *y* values. As explained earlier, in an orthogonal coordinate system, the *X*-axis and the *Y*-axis are perpendicular to each other and divide the plane into four parts. The four parts are called quadrant I, quadrant II, quadrant III, and quadrant IV. In quadrant I, $x > 0, y > 0$; in quadrant II, $x < 0, y > 0$; in quadrant III, $x < 0, y < 0$; and in quadrant IV, $x > 0, y < 0$

The piece in the game "dam-dam" can be seen as a point, and the intersections between the lines become the position of the points. Figure 2 shows that the sample has a similarity in which at least there are two points in quadrant I with the pieces' initial position. The two points are considered as point A and point B.

Point A and point B are points where the *x* and *y* values are positive, meaning that point A and point B are requested to be in quadrant I. Besides, point A and point B both have different ordinates and abscissa. Thus, it shows the position of point $A(x_1, y_1)$ and point $B(x_2, y_2)$.

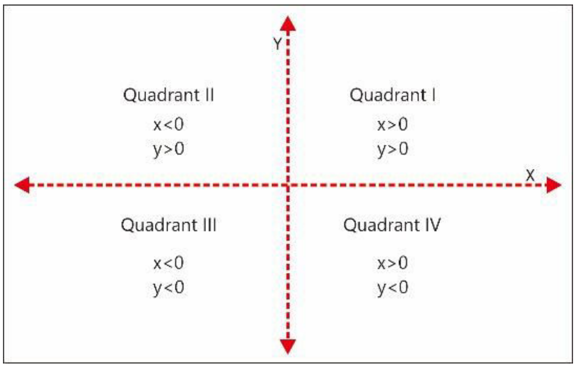

Figure 4. Orthogonal coordinate system

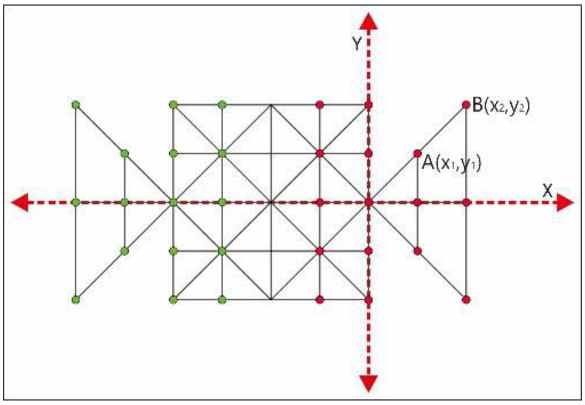

Figure 5. Sample of the position of point A and point B

Distance between Two Points

After knowing point A and point B, then the distance between the two points can be identified.

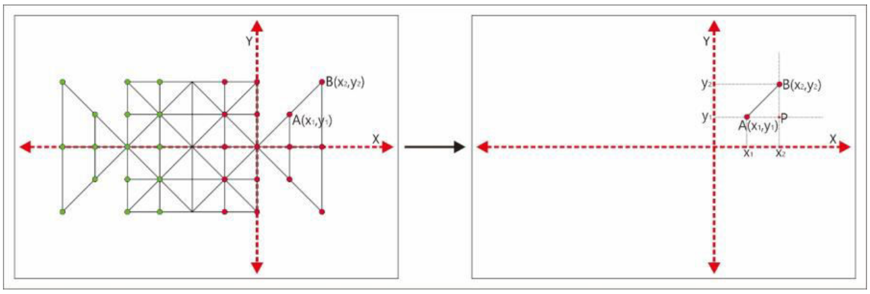

Figure 6. Determine the distance between point A and point

Through point A, draw a parallel line to the *X*-axis, and through point B, draw a parallel line to *Y*-axis; both lines intersect at point P and form ΔAPB which is a right triangle.

Length of the lines $|AP| = |x_2 - x_1|$ (12)

Length of the lines $|BP| = |y_2 - y_1|$ (13)

Length of the lines $|AB|$ (the distance between the two points identified) can be identified using the Pythagorean theorem.

Theorem 2 (Pythagoras theorem)
"In a right triangle, the hypotenuse of the square is equal to the sum of the squares of the other sides."
Then:

$$|AB|^2 = |AP|^2 + |BP|^2 \quad (14)$$

Substitution (8) and (9)

$$|AB|^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

$$|AB| = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2} \quad (15)$$

Theorem 3
The distance between points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$|AB| = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

Equation of a Straight Line through the Origin Point with the Gradient m

Point projection $A(x_1, y_1)$ to X-axis is $P(x_1, 0)$

Point projection $B(x_2, y_2)$ to X-axis is $Q(x_2, 0)$

Pay attention ΔOQB with $AP \parallel BQ$, then:

$$|AP|:|OP| = |BQ|:|OQ| \quad (16)$$

$$y_1 : x_1 = y_2 : x_2 \quad (17)$$

$$\frac{y_1}{x_1} = \frac{y_2}{x_2}$$

$$y_1 = \frac{y_2}{x_2} x_1 \quad (18)$$

Because α is an angle formed by line l with the X-axis in the positive direction, then:

$$\frac{y_1}{x_1} = \frac{y_2}{x_2} = \tan\alpha \quad (19)$$

It appears that the ratio of the ordinate and the abscissa of each point on the line l is $\tan\alpha$. If points (x, y) are in line l, then:

$$\tan\alpha = \frac{y}{x} \quad (20)$$

Because point $B(x_2, y_2)$ has been identified, then the value of certain $\tan\alpha$ is:

$$\tan\alpha = \frac{y_2}{x_2} \quad (21)$$

It obtains:

$$\frac{y}{x} = \frac{y_2}{x_2}$$

$$y = \frac{y_2}{x_2} x$$

Thus, the equation for the straight line l through the origin point O and $B(x_2, y_2)$ is:

$$y = \frac{y_2}{x_2} x \quad (22)$$

α is the angle formed by line l with X-axis with positive direction

$\tan\alpha$ is called the gradient/coefficient of the direction of line l, and typically given the symbol m. So, $m = \tan\alpha$.

Thus, the equation for the straight line l that passes through the origin point O with the gradient m is $y = mx$.

Theorem 4
The equation of a straight line that passes through

the origin point $O(0{,}0)$ and point $P(x_1, y_1)$ is:
$y = \frac{y_1}{x_1}x$.

Theorem 5
The equation for the straight line through the origin point $O(0{,}0)$ with the gradient m is: $y = mx$, with $m = \tan\alpha$.

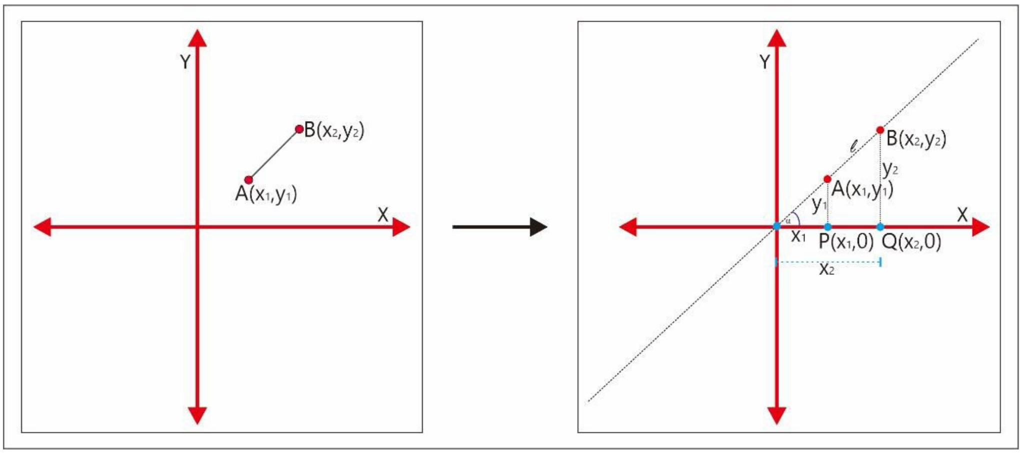

Figure 7. Determine the equation of the straight line with a gradient (m)

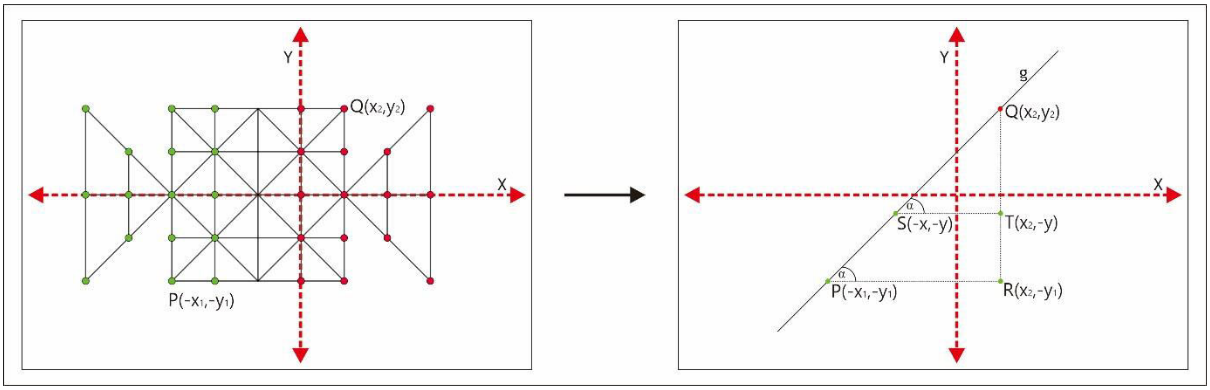

Figure 8. Determine the equation of the straight line through two points

Equation of Straight Line through Two Points

After knowing the straight-line equation through the origin point O with the gradient (m), the straight-line equation can be identified through two points. In the game "dam-dam," many lines pass some points. Take any point $P(-x_1, -y_1)$ and $Q(x_2, y_2)$ In the game, then, it can show the equation of the line formed.
First, determine the gradient of line g, namely, $\tan\alpha$
Pay attention to ΔPRQ, which is a right triangle with $\angle QPR = \alpha$, because PR is parallel with X-axis and QR is parallel with Y-axis, it means:
$|PR| = x_2 + x_1$ and $|QR| = y_2 + y_1$ so:

$$\tan\alpha = \frac{|QR|}{|PR|} = \frac{y_2 + y_1}{x_2 + x_1} \qquad (23)$$

Take any point $S(-x, -y)$ on the line g, shows the intersection point $T(x_2, -y)$, then the gradient of line PQ is the same as the gradient line QS. While the gradient line QS is:

$$\tan\alpha = \frac{|QT|}{|ST|} = \frac{-y - y_2}{-x - x_2} = \frac{-(y + y_2)}{-(x + x^2)} \qquad (24)$$

Because (23) is the same with (24), then:

$$\frac{-(y + y_2)}{-(x + x^2)} = \frac{y_2 + y_1}{x_2 + x_1} \qquad (25)$$

$$-\frac{y + y_2}{y_1 + y_2} = -\frac{x + x_2}{x_1 + x_2} \qquad (26)$$

Equation (26) is the equation of a straight line through two points $P(-x_1, -y_1)$ and $Q(x_2, y_2)$, then the equation of the straight line through two points $P(x_1,y_1)$ and $Q(x_2,y_2)$ is

$$\frac{y+y_2}{y_1+y_2}=\frac{x+x_2}{x_1+x_2} \quad (27)$$

Theorem 6

The equation of the straight line through two points $P(x_1,y_1)$ and $Q(x_2,y_2)$ is $\frac{y+y_2}{y_1+y_2}=\frac{x+x_2}{x_1+x_2}$

Methodology

This research and development used the R & D stage proposed by the Plomp. This model consists of the initial investigation, design, construction (realization), trial (test, evaluation, and revision), and implementation (Fernandes et al., 2020). Research and development obtain valid, practical, and effective learning materials (Badri & Yerizon, 2021). This research was conducted at one of the famous universities in Central Java involving students of the mathematics education study program who are currently taking the analytic geometry course. Two experts in analytic geometry assessed the validity of the material. The practicality of the material was based on the subjects' response in using the developed product. Then, the material's effectiveness was to see whether the material can effectively increase students' understanding. The effectiveness test uses the independent t-test by comparing the experimental class and the control class (Suciati, Maridi, Dewi, Subandowo, & Sasmito, 2020). This effectiveness used the nonequivalent control group design (Wicaksana, Widoretno, & Dwiastuti, 2020). The research design is presented in Table 2.

Table 2. Nonequivalent Control Group Design

E	O_1	X	O_3
C	O_2		O_4

E = experimental class
C = control class
X = treatment by giving the developed material
O_1 = Pre-test of experimental class
O_2 = Pre-test of the control class
O_3 = Post-test of experimental class
O_4 = Post-test of the control class

The pre-test was conducted to obtain data of the students' initial abilities from the two groups. The post-test for the experimental class was conducted to obtain data of the students' final abilities after using the developed material. The instruments for both pre-test and post-test were first tested for their validity and reliability (Sukardi, Prihono, Syahruzah, & Maulana, 2020), and the result is valid and reliable.

Results

The materials' development covered five stages, namely, initial investigation, design, construction (realization), testing (test, evaluation, and revision), and implementation.

Initial Investigation

The initial investigation includes observation, literature study, and needs analysis (Pudjastawa et al., 2020). The results showed that there are gaps between field conditions and learning ideals resulting in low learning achievement. It is indicated by the high percentage of students who did not pass the final examination due to low students' interest in learning, limited learning materials, and use of the online learning system.

Design

The researcher designed learning materials and the research instruments required (Wahyuni & Kurniawati, 2021). The design of research instruments refers to the results of the initial investigation stage. The material was designed based on the development's objective to get valid, practical, and effective material. The target users are students of the mathematics education study program.

Construction (Realization)

This stage is a follow-up to the design stage. Learning multimedia and the instruments needed are constructed or developed (Rusdi et al., 2020).

Testing

Testing is to see whether the developed product can achieve its goals and objectives or not

(Samaniego & Figueres, 2020). The product was tested twice for expert judgment and limited-scale trials. The product was validated by experts using a validation sheet with five alternative responses, as presented in Table 3 below.

Table 3. The Assessment Score Basis for Expert Judgment

Criteria	Score
Very good	5
Good	4
Quite good	3
Less good	2
Poor	1

The data obtained from experts were in the form of qualitative data; therefore, they were converted into quantitative data before further analysis. The validity of the material was assessed by referring to the Benchmark Reference Assessment (PAP) proposed by Budiyono (2020: 141) as presented in Table 4.

Table 4. Modified Mean Score Conversion

Interval	Category
$1.00 \leq \bar{X} < 1.80$	Not valid
$1.80 \leq \bar{X} < 2.60$	Less valid
$2.60 \leq \bar{X} < 3.40$	Quite Valid
$3.40 \leq \bar{X} < 4.20$	Valid
$4.20 \leq \bar{X} < 5.00$	Very Valid

The results of the two experts' assessment showed a mean value of 3.90, indicating that the developed material is valid.

The developed learning material is considered practical if students state that the material can be used properly. The learning material's level of practicality was obtained from students' responses to the closed questionnaire after using the developed material. The questionnaire consisted of questions about using learning materials using a Likert scale with specific assessment criteria. The questionnaire covered items with positive and negative statements. The criteria for assessing the practicality used PAP proposed by Budiyono (2020: 141) as presented in Table 5.

Table 5. Criteria for Practicality Assessment Using PAP

Value (%)	Criteria of Practicality
$80.0 < P \leq 100$	Very practical
$60.0 < P \leq 80.0$	Practical
$40.0 < P \leq 60.0$	Quite practical
$20.0 < P \leq 40.0$	Less practical
$0.00 \leq P \leq 20.0$	Not practical

Based on the results of the students' responses to the questionnaire, the percentage of the practicality of the material reached 78.0%, indicating that the developed material is practical.

The effectiveness of the developed material was analyzed using statistical analysis. The data used met the statistical analysis requirements by checking the normality and homogeneity of the pre-test and post-test results, both in the control group and in the experimental group. The normality and homogeneity test used SPSS 25 software. The normality of the data was tested using the Kolmogorov-Smirnov and Shapiro-Wilk. It showed that the pre-test and post-test data in the experimental and control groups were generally distributed, as presented in Figure 8 and Figure 9.

Tests of Normality

		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Class	Statistic	df	Sig.	Statistic	df	Sig.
Pre-test	Experiment	.094	38	.200*	.981	38	.756
	Control	.143	36	.062	.970	36	.431

*. This is a lower bound of the true significance.
a. Lilliefors Significance Correction

Figure 8. Results of Tests of Normality for Pre-Test

Tests of Normality

		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Class	Statistic	df	Sig.	Statistic	df	Sig.
Postest	Experiment	.108	38	.200*	.952	38	.101
	Control	.129	36	.140	.976	36	.624

*. This is a lower bound of the true significance.
a. Lilliefors Significance Correction

Figure 9. Results of Tests of Normality for Post-Test

The homogeneity test for the pre-test and post-test of the control and experimental groups was performed using the Levene test. The results showed that both the pre-test and post-test data were homogeneous or came from the same population. The pre-test data were to determine whether the two groups (experimental and control) had the same initial ability and found out whether the developed material can significantly increase students' understanding based on the results of the independent t-test post-test data of sig. the value of 0.017 with an average value of 51.89 for the experimental group and 47.91 for the control group. The proposed hypothesis is the experimental group has a better understanding than the control group. The test results obtained the sig. value < significance level of, based on the 5% significance level, the hypothesis is accepted. Therefore, the developed material effectively improves students' understanding of the material of point and line position.

Implementation

Implementation can be done by publishing the results of development in scientific forums and through publication in scientific journals, as well as the use of learning materials in the field (Handayani et al., 2020) This study was distributed to students of mathematics education study program.

Discussions

Based on the findings above, the developed material is valid, practical, and effective. The material is declared valid by the expert as in its preparation is based on relevant rules and theories. It shows that through the ethnomathematics approach, the learning carried out can improve student's understanding. Herawaty et al. (2020) state that the students' thinking process in understanding parallel axioms through ethnomathematics learning is abstract activities of remembering, understanding, analysing, assessing, imagining, reasoning, and speaking in learning geometry. Besides, the ethnomathematics-based learning resources can help create meaningful learning (Vitoria & Monawati, 2020) and look real and exciting (Fauzan et al., 2020).

The concept of ethnomathematics is an innovation in mathematics learning (Supriadi et al., 2019). Learning innovation by utilizing the ethnomathematics concept can be performed in many aspects. For example, Rohaeti et al. (2020) developed an ethnomathematics-based learning model. On the other hand, ethnomathematics-based learning can support the development of students' geometric thinking skills as the existing culture requires geometric shapes Deda & Amsikan (2019); Supiyati et al. (2019); Wijayanti et al. (2019) have conducted ethnomathematics research on geometry.

The material developed in this study was integrated with van Hiele's five learning phases. Therefore, the increase in students' understanding of the material cannot be separated from van Hiele's learning phase implementation. Some previous studies found the same results, for example, by Hassan et al. (2020); Machisi (2021); Machisi & Feza (2021), Pasani (2019). Thus, it can be said that conclusively van Hiele's geometry learning theory is proven effective in teaching and learning mathematics. The five phases are information, guided orientation, explication, free orientation, and integration (Sari et al., 2020).

Conclusion

Based on the above discussion, it can be concluded that the developed material is valid, practical, and effective. Students' understanding increases after using the material. The results showed that the integration of ethnomathematics in mathematics learning and the use of van-Hiele's learning phases should be considered to maximize the student learning process. Thus, it is expected that lecturers of Mathematics facilitate students with proper learning materials and integrate ethnomathematics in mathematics learning and apply van-Hiele's learning phase in the learning process at the university.

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