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How is the Curriculum (Courseware) Effective in Increasing Students' Level of Understanding of Three-Dimensional Concepts Using APOS Theory?

Vahid Alamian[1], Masoomeh Noori[2]
[1]Faculty member, Department of Mathematics, Farhangian University, Iran-Tehran
E-Mail:vahid_alamian@yahoo.com
[2]Math teacher ,Tehran Educatin

ABSTRACT

This study aims to examine the effect of the GeoGebra environment on increasing the students' understanding of three-dimensional concepts (R^3) based on APOS theory. The present study is quasi-experimental and includes two experimental and control groups. Pre-test and post-test are applied in both of the groups. The statistical population consists of 324 female students of the 12th-year mathematics and physics in the Tehran 15th district in 2020. The sample includes a class of 30 individuals and another class of 31, which have been homogeneously divided from the beginning of the year, and one of the classes is randomly selected as the experimental group and the other as the control group. According to the lesson plan of working with GeoGebra courseware, the three-dimensional space concepts are taught for 60 minutes and in 3 sessions in the experimental group. However, according to the lesson plan, the student's problems are reviewed and resolved in the control group in these 3 sessions. First, a researcher-made test with 16 questions (4 questions from each level) is prepared, and CVR is used to check the validity of the questions. Then, a test with 4 selected questions is used to identify students' problems, and the reliability is obtained using Cronbach's alpha method, and the percentage of students' correct answers at each level was identified by the APOS method. Data analysis was performed using SPSS24 software at two descriptive and inferential statistics levels. According to the results, which showed an increase in students' level of understanding at all levels of APOS theory and a decrease in zero level (no answer sheets), it can be concluded that education using GeoGebra courseware can improve students' level of understanding.

Keywords
Geometry Education, GeoGebra Environment, Students' Understanding, Three-Dimensional Space, APOS Theory

Introduction

Nowadays, the competencies and skills of teaching mathematics are the main influential elements on the progress of any nation, especially in science and technology. The fundamental importance of mathematics for human beings can be expressed through the interrelationship between mathematics and human development to improve human goals (P.S.Madonsela, 2020). Many educational problems in mathematics, including students' reluctance and the negative attitude of many adults towards mathematics, especially geometry, and the problems of many graduates and their inability to imagine the concepts of geometry, are the results of the traditional teaching method. Many students have problems understanding geometric and mathematical concepts such as working with three-dimensional cubes, two-dimensional images, making three-dimensional objects, and recognizing and comparing three-dimensional geometric volumes (Zanganeh and Saedi, 2016).

While acknowledging the importance of geometry, many studies have emphasized the difficulties in teaching geometry and being unsuccessful in achieving the desired goals. Geometry is one of the most difficult branches of mathematics that students face. This point shows the reason for being unsuccessful in this field (Mohammad Ahmad Alkhateeb, 2019). The important point for understanding geometry at higher levels is that the students' need to learn concepts depends on the ability to understand geometry, and also the development of people's minds in solving geometry problems cannot be denied. All these reasons and many other reasons emphasize that students' views on this course should be changed, and this is not possible except by applying appropriate methods to understand and solve geometry problems with useful tools (Zamani, 2018).

In mathematics, it is important to understand the three-dimensional space concept. Although three-dimensional space is a visible space, the concepts of point coordinates and the linear equations and plane and image and asymmetry and 8 corners of R^3 are unimaginable, especially for female students, because generally, due to inherent characteristics, women are weak in three-dimensional and spatial perceptions (Dejdar,2016).

We cannot guarantee complete learning of a subject just by talking about it in mathematics education. According to Dale's cone of experience, we remember 30% of what we hear but 80% of what we see, hear and speak. Using GeoGebra, students become more involved in the teaching-learning process, and more sensory members are involved (Lal Kumarsink, 2018). Special features of GeoGebra software in displaying three-dimensional space, especially algebraic and geometric windows together, the ability to rotate images from all dimensions, and the availability of new versions for everyone are the reasons for using this software to better understand the basic concepts of three-dimensional space and to solve the spatial perceptions problems in the students' minds. The software also introduces a new way of learning math, attracting students' interest. This software attracts students' attention in teaching and focuses on problem-solving. Therefore, GeoGebra provides a good environment for learning mathematics. Researchers find that the program, with its high accuracy in graphic representation, the performance of geometric transformations, controlling the drawing of curves, showing

various connections, and feedback, help the students to easily correct their work (Mohammad Ahmad Alkhateeb, 2019).
It should be noted that a study in Jakarta, Indonesia, on 12^{th}-grade students shows that learning three-dimensional geometry with GeoGebra is more effective for students with previous high and medium math skills, and in the group of students with previous low math skills does not produce a significant effect (Yaya S. Kusumah, 2020). APOS theory as a learning theory can classify students 'problems and evaluate their learning from the three-dimensional space concepts. APOS is a learning theory in mathematics. It analyzes the mental structures of an individual in a mathematical concept (Arnon et al., 2014). APOS theory comprises mental structures and includes actions, processes, objects, and schemas (Burji et al., 2018). Thus, this study aims to investigate the effect of produced courseware in the GeoGebra environment on increasing the students' understanding of three-dimensional space $(\mathbb{R}^3)$ concepts using APOS theory.

Theoretical Foundations of Research

Three-Dimensional Geometry and Three-Dimensional Space

Spatial abilities are the type of mental activities that enable individuals to create spatial images and manipulate them to solve various practical and theoretical problems (Hegarty & Waller, 2005; Kozhevnikov ‹Motes & Hegarty, 2007). Three-dimensional geometric abilities include relevant knowledge and skills such as grid construction, displaying three-dimensional objects with two-dimensional shapes, identifying solids and their elements, constructing cubic arrays, calculating the surface and volume of solids, and comparing the properties of three-dimensional shapes (National Council of Mathematics Teachers, 2000). Improving students' spatial ability is emphasized in teaching three-dimensional geometry (Clementes and Saraya, 2007; Clementes and Batista, 1992; Gutierrez, 1996; Persmeg, 2006). The three-dimensional geometry knowledge and the "sense of space" are emphasized by the Math Teachers' Council, which further states that students should acquire the first three levels of Van Hiele through the K-12 curriculum. However, why do we need to improve children's "sense of space," especially in math classes? Spatial ability and mathematical progress are related. Although we do not completely understand why and how it works, children with strong spatial senses are better at math (Clementes 2004). To have a sense of space, learners need spatial abilities. The two main abilities are spatial orientation and spatial visualization (Bishop, 1980).
As Freudental said, spatial geometry and reasoning are inherently important because they involve "learning the space in which the child lives, breathes, and moves ... the space that the child must learn to know, to discover, to conquer, to live and breathe there and move better" (Clementes, 2004). Patelis and Cristo (2010) showed that the spatial abilities with the reasoning in three-dimensional geometry suggest that three-dimensional geometry training should develop spatial abilities.

APOS Theory

The concept construction in this theory is like a cycle that occurs in the learner's mind. APOS theory which was based on one of Piaget's theories, was proposed by Dubinski in 1991. Dubinski reconstructed it in academic mathematics (Osilia et al., 1997). The structures in APOS theory are action, process, object, and schema. In other words, APOS theory starts with actions, moves through processes, and gets to schemas after turning them into objects (Tall, 1999).

Action

The action changes the known objects to the person, but the changes should be shown to the learner. At this stage, the learner explicitly needs his memory to do tasks and problems step by step (Vinner, 2010).
In the definition of the mental structures of action, Asiala et al. (1996) state that understanding a mathematical concept begins with manipulating previously constructed mental or physical objects that give rise to actions (Renon et al. 2014).
Based on Piaget's theory derived from APOS theory, Arnon et al. (2014) believed that a concept is first perceived as an action (an externally directed evolution of an object or objects previously perceived). Arnon et al. (2014) state that it is external action, meaning that external instructions must explicitly guide each stage of evolution. In addition, each stage triggers the next stage, and therefore the person has no idea of the stages of action and also cannot leave those stages and ignore them. A person who is limited in action is dependent on external guidance.

Process

When an action is repeated, and the person reflects on it, it becomes an internal process. At this time, the learner has an internal structure to act. It means that internalization allows a person to become aware of the action, combine it with other actions, or coordinate two or more processes to create a new process. Processes have the same internal action structure, but they do not need guidance from external stimuli. A person who understands the change process can reflect on the action, describe the action, and even reverse the steps of change without actually taking those steps. Compared to action, the process is no longer guided by external guides and stimuli but by the learner (Khoshnood, quoted in Nazari, 2011).
Arnon et al. (2014, quoting from Dubinski et al., 2005) provide the following description of the process; When a person repeats and reflects on an action, that action may become internalized in a mental process. The process is a mental structure and performs the same as an internalized action, but completely in the individual mind. So the individual thinks he can do the transformation without performing all the steps explicitly.

Object

When a person knows the process as a whole and realizes that he can change actions and can make such changes, his thinking is changed from the process level to the object level, or the process summed up in the object. When performing an action or process on an object, extending the object to the processes from which it is derived (Khoshnood, quoted in Chamanara, 2018). Arnon et al. (2014) state that to turn the process into the object and object to processing, three mental mechanisms of encapsulation, decapsulation, and coordination are needed in the student's mind.

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Encapsulation occurs when an individual uses an action for a process. In other words, it sees a dynamic structure (process) as a fixed structure so that action can be applied. Dubinsky et al. (2005) explain that if one becomes aware of the process as a whole, he can activate the change based on that whole, and in fact, that process can bring about such changes (explicitly or in the individual's mind). Therefore, it can be said that one encapsulates the process in a cognitive object. Studying various research about APOS theory shows that the encapsulation mechanism is very difficult, and in most of these studies, a small number of students have reached this stage (Arnon et al., 2014). A coordination mechanism is necessary to construct some objects. Two objects can be decapsulated to reach their process. Then, it is possible to coordinate these processes and encapsulate them to form a new object (Arnon et al., 2014). Matenga Bernard (2016) says that a student with a cognitive object can use it as a new action, apply it in other fields, find its properties, or link it with other made objects.

Schema

When actions, processes, and objects are created, they can be intertwined differently. For example, two or more processes may be interconnected by combination or other ways. In other words, a set of processes and objects in the type of construction are organized as a schema. On the other hand, one can reflect on the schemas and act on them, which leads to creating a new object. Therefore, objects are made in two ways. i.e., process and schemas (Khoshnood, quoted by Sharifi, 2018). The idea of schema is very similar to Tall and Vinner's idea of conceptualization. They believed that conceptual imagery provides a concept for a person, such as diagrams, symbols, and verbal representations of numerical information and a set of properties. Conceptual perception is formed over the years through different experiences and changes and gets more complete in the face of new stimuli. This idea is opposed to the previous definition of the concept (Khoshnood, quoted by Mohtasham, 2018).

Research Method

The research design is semi-experimental (quasi-experimental). It includes two experimental and control groups and pre-test and post-test in both of them. The statistical population consists of 324 female students of the 12^{th}-year mathematics and physics in the Tehran 15th district in 2020. It is a sample of 61 people (students of two classes) from Tarbiat High School. The samples of this study are a class of 30 people and a class of 31 people, which have been homogeneously divided from the beginning of the year, and one is selected randomly as the experimental group and the other as the control group.

In the experimental group, according to the lesson plan (software installation steps, familiarity with the software environment, familiarity with the required options of GeoGebra, use of algebraic and graphical windows to display and understand and solve concepts familiar with three-dimensional space), working with GeoGebra software in the three-dimensional space concept is taught for 3 sessions and 60 minutes. However, in the control group, according to the lesson plan (solving pre-test questions by students themselves in WhatsApp 6-person groups - solving similar problems in student groups and discussion and conclusion in WhatsApp 6-person groups - reviewing major problems in the Skyroom under the control of the teacher), the student's problems are solved. Finally, a post-test is performed between the two groups, and their performance is measured about each other.

First, a researcher-made test with 16 questions (4 questions from each level) is prepared, and CVR is used to check the validity of the questions. Then a test with 4 selected questions is performed to identify students' problems, and the reliability is obtained through Cronbach's alpha method, and the percentage of correct answers of students in each of the APOS levels is identified with this test.

To determine the reliability of pre-test and post-test, 8 questions were selected in the previous stage with the appropriate value of CVR. Then they were given twice to 25 12th-grade students other than the sample school, and after checking the papers, descriptive scoring was done in five categories of excellent, good, average, poor, and very poor for each question. By assigning numbers from 1 to 5, the answers were categorized and quantified on the Likert scale. After that, Cronbach's alpha value was calculated using SPSS software. Finally, the alpha value was 0.796 for the pre-test questions and 0.824 for the post-test questions, and considering that the alpha value in both tests was more than 0.7, the reliability was also confirmed.

Given that pre-test and post-test questions are designed to measure students' level of understanding based on APOS theory; thus the related indicators should be predicted to outline questions, which these indicators are extracted based on the research algebraic thinking skills (Fevi Rahmawati Suwanto, 2017), and are indicated in Table (1) along with the examples of each index.

Table 1. Pre-test and Post-test Questions to Measure Students' Level of Understanding based on APOS Theory

APOS level	Indicators of APOS level	Question	Study the question
Action	Activities are related to procedural matters. Focus on the algorithm in solving the problem. According to the example, try to solve the problem. Only according to mathematical concepts, use formal formulas. It requires precise guidance to solve the	If we have A(-1,3,-2), B(1,-3,-2), calculate the midpoint coordinates of the line segment AB (pre-test).	To solve this question, it is enough for the student to know the formula for the midpoint coordinates and calculate the midpoint coordinates by placing the given coordinates.
		If, B (3, -1, 6), A (1, - 3, 2). Obtain the segment length AB (post-test).	The student must use the formula to calculate the length of a line segment or the same distance between two points and place the given coordinates; Calculate the length of the line segment.

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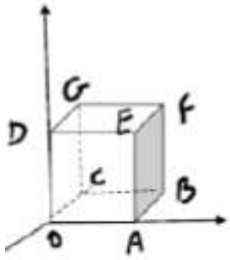
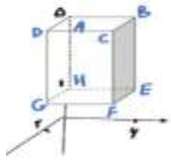
	problem (transformation).		
Process	It does not need help to convert. Be able to explain the structured stages of transformation without actually doing it. This activity is a procedural understanding.	Consider point A (2, -3,4). A) Name the symmetry of point A concerning the YOZ plane of point A' And obtain its coordinates. B) Name the image of point A concerning the axis Z of point and obtain its properties. (Pre-test)	To answer part (A), the student must mentally (internalize) point A and YOZ page, find the beginning of the image A on this page, and then, considering this page as the axis of symmetry, obtain the symmetry of this point. To answer part b, the student must mentally find the image of point A on this axis by imagining point A and axis Z.
		A) Write the image coordinates of point A (2, 1,4) on the plane Z = 2. B) Write the symmetric coordinates of point A (2, 1,4) concerning plane Z = 2. (Post-test question).	To answer part A of this question, the student must mentally (internalize) point A and page 2 = Z to find point A's image on this plane. To answer part B of this question, the student must mentally (internalize) point A and plane 2 = Z, find the beginning of image A on this page, and then, considering this plane as the axis of symmetry, obtain the symmetry of this point.
Object	Perform actions and processes against encountered mathematical objects. This process is a conceptual understanding. Able to explain logically and structurally the performed transformation. Be able to explain the properties of mathematical concepts.	Pre-test question The provided figure shows a cube in the coordination of R^3. If the coordination of the point is C(-2,0,0), then: A)The vertex coordinates of F. B)ABEF face equation. C) GF edge equation 	To answer part A of this question, the student must use the given coordinates of point C and the dimensions of the cube to be equal to the equations of the constituent planes of this cube internally. Then, given that point F is the intersection of three planes of X=2, Y=2, Z=2, write the coordinates of the point F (part A of this question is at the process level, but this question is included.) Part B should provide a conceptual understanding of the aspect as a finite plane, explain the properties of the points in this aspect, and finally write the properties of the concept of this aspect in mathematical language. In part C, one must understand the edge concept as a limited line. Explain the properties of the points on this line segment and write the properties of the concept of this edge in mathematical language.
		Post-test question In this rectangular cube, find: A)The vertex coordinates of E B)The edge equation of BC C) The face equation of DCGF 	To answer part A of this question, the student must use the coordinates specified in the figure to arrive at the equations of the constituent planes of this rectangular cube internally. Then, given that point E is the intersection of three planes Z = 1, Y = 6, X=o, write the coordinates of point E (this part is at the level of the process). Part B should provide a conceptual understanding of the aspect as a finite plane. Explain the properties of the points in this aspect.

			Finally, write the properties of the concept of this aspect in mathematical language. In part B, one must understand the edge concept as a limited line. Explain the properties of the points on this line segment. Finally, write the properties of the meaning of this edge in mathematical language.
Schema	Connect different concepts of mathematics (Evolution of concepts about action activities, processes, objects can relate actions, processes, objects to other previously known items). Understand the necessary formulas to solve the problem.	Draw a rectangular cube from the collision of the main planes with the planes x=a, y=b, z=c. Then prove that the diameter of this rectangular cube is equal to $\sqrt{a^2+b^2+c^2}$ (pre-test)	To answer this question, we must first draw this rectangular cube using the equations of the given planes and the main planes. Then, recalling the concept of the diameter of a rectangular cube, draw a diameter for this rectangular cube using the formula for the length of a line segment and the coordinates of two rectangles, or using right-angled triangles and the Pythagorean formula to calculate the diameter.
		Consider point A (x, y, z). Prove by drawing a figure that the distance from a point on the z-axis is equal to $\sqrt{x^2+y^2}$ (post-test)	To answer this question, we must first draw point A with the given coordinates in space R^3. Then draw the image of this point on the Z-axis, write its coordinates, and then calculate the distance of point A from the Z-axis using the two-point distance formula.

Data analysis of this study was performed using SPSS24 software at two levels of descriptive and inferential statistics. At the level of inferential statistics, Levene's test (Fisher) was used to check the similarity of variances, and a t-test was used to compare the performance of the two groups and determine the significance level.

Research Questions

Main question: How the use of produced curriculum (courseware) is effective in increasing the level of students' understanding of three-dimensional space $(\mathbb{R}^3)$ based on APOS theory?

Sub-question 1: What is the students' understanding of the subject $\mathbb{R}^3$ space in the framework of APOS theory based on the pre-test results at the action level?

Sub-question 2: What is the students' perception of $\mathbb{R}^3$ space in the framework of APOS theory based on the pre-test results at the process level?

Sub-question 3: What is the students' understanding of $\mathbb{R}^3$ space in the framework of APOS theory based on the pre-test results at the object level?

Sub-question 4: What is the students' understanding of $\mathbb{R}^3$ space in the framework of APOS theory based on the pre-test results at the schema level?

Sub-question 5: What is the effect of using produced courseware in the GeoGebra environment on increasing students' understanding of $\mathbb{R}^3$ space in practice?

Sub-question 6: What is the effect of using the produced courseware in the GeoGebra environment on increasing students' understanding of $\mathbb{R}^3$ space at the process level?

Sub-question 7: What is the effect of using the generated curriculum (courseware) in the GeoGebra environment on increasing students' understanding of $\mathbb{R}^3$ space at the object level?

Sub-question8: What is the effect of using the produced courseware in the GeoGebra environment on increasing students' understanding of $\mathbb{R}^3$ at the schema level?

Findings

To determine the level of students 'understanding based on APOS theory, the correct answer to the highest number of questions was used as a leveling criterion, and to show numerically the level of students' understanding based on APOS theory, the number 0 was given to students who do not fall into any of the levels (did not answer any questions correctly). The numbers 1, 2, 3, and 4 were used for the students at the APOS theory levels, i.e., action, process, object, and schema, respectively. It should be noted that if the process level questions are solved using the formulas in the test books and classes, such answers were ultimately considered at the action level. For this purpose, students were asked to explain their solution method in writing.

Table 2. Number and percentage (relative to the whole class) of students who answered the questions of each level of APOS theory correctly in the pre-test

	Control group					Experimental group				
Type of level	Action	Process	Object	Schema	Zero level	Action	Process	Object	Schema	Zero level
Frequency										
Percentage										

Table 3. Number and percentage of students (relative to total) by the level at each level of APOS theory in the pre-test

	Control group					Experimental group				
Type of level	Action	Process	Object	Schema	Zero level	Action	Process	Object	Schema	Zero level
Frequency										
Percentage										

Table (2) shows that 80% of the students in the control group and 77.4% of the students in the experimental group were able to answer the practice (action) level questions correctly in the pre-test, this happens for the control group at the process level to 3.43% and the experiment to 42%, and at the object, the level is 20% and 16.1%, respectively. 6.4% of the students in the experimental group and 6.7% of the students in the control group were able to answer the schema level questions, and 20% of the students in the control group and 22.6% of the students in the experimental group were not able to answer any of the pre-test questions and placed at zero level.

Table (3) shows the number and percentage of students who have been tested on only one level of APOS theory. According to this table, for the control group in the pre-test, the highest percentage of students was at the action level (36.7), then the process level (23.3), zero (20), and object (13.3), and 6.7% at schema level schema. The same thing happened to the experimental group.

Table 4. Number and percentage (of the total class) of students who answered the questions of each level of APOS theory correctly in the post-test

	Control group					Experimental group				
Type of level	Action	Process	Object	Schema	Zero level	Action	Process	Object	Schema	Zero level
Frequency										
Percentage										

Table (4) shows that 86.7% of the students in the control group and 96.7% of the students in the experimental group were able to answer the action level questions correctly in the post-test. This is 80% at the process level for the control group, 83.8% for the experimental group, and 36.7% and 67.7% at the object level, respectively. Three students in the control group answered 10% of the schema level questions correctly. Five students in the experimental group, i.e., 16.1%, answered the schema level questions correctly. And 13.3% of the students in the control group and only one person, i.e., 3.2%, of the students in the experimental group did not answer any of the post-test questions.

Table 5. Number and percentage of students (relative to total) disaggregated at each level of APOS theory in post-test

	Control group					Experimental group				
Type of level	Action	Process	Object	Schema	Zero level	Action	Process	Object	Schema	Zero level
Frequency										
Percentage										

Table (5) shows the number and percentage of students who have passed the exam at only one level of APOS theory. According to this table, for the control group in the post-test, the highest percentage (43.3) of students is at the process level and then at the object level (26.7), zero (13.3), schema (10), and action (6.7). However, the highest percentage (51.6) of the experimental group students were at the object level; 16.6% of the students were at the process level, 16.6% were at the schema level, and 13% were at the action level.

Table 6. Statistical indicators of students' level of understanding according to APOS theory

Statistical indicators	Pre-test of the control group	Pre-test of the experimental group	Post-test of the control group	Post-test of the experimental group
Mean				
Variance				
Standard deviation				

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Mode				

According to Table (6), it can be concluded that the average level of students' understanding in the experimental group based on APOS theory with the effect of problem-solving is 2.65, which is increased in comparison with 1.16 pre-test.
After determining the control group and the experimental group, the first and most necessary task is to examine the similarity of the variances of the two groups. For this purpose, a pre-test was performed on both groups. The results were analyzed using Levene's test in SPSS software. The results are shown in Table (7).

Table 7. Evaluation of variance of experimental group and control group in pre-test based on Levene's test

Variable	Levene's statistic (Fisher)	The first freedom degree	The second freedom degree	Significance level
Students' level of understanding based on APOS theory	0.093	0.956	59	0.761

In Levene's test, H_0 is the equality of two groups variances, and about the significance level of 0.761 in Table (7) and since this number is greater than 0.05, so the test hypothesis, which is the equality of the variances of the experimental and control groups, is confirmed.

Table 8. Results of independent t-test to evaluate the significance of the differences between the data of the control group and the experimental group in the post-test

Variable	Levene's statistic (Fisher)	The first freedom degree	The second freedom degree	Significance level
Students' level of understanding based on APOS theory	0.659	0.171	59	0.42

In the independent t-test, the H_0 hypothesis is that there is no significant difference between the data of the control and the experimental groups in the post-test. However, according to the results of Table 4-11, the significance level of this test is 0.42, and this value is more than 0.05, so hypothesis H0 is confirmed, and there is a significant difference between the data of the control group and the experimental group in the post-test. It seems that three makeup class sessions for the control group, which were mostly based on group work and solving questions and problems in groups of 6 with the cooperation of the students themselves, was effective in learning, and there was no significant difference in the post-test results of the control and experimental groups. However, the high number of correct answers of the experimental group students at the object level shows the positive effect of using software in teaching because, in the control group, this superiority is seen at the process level, which requires a lower level than the object level.
Furthermore, due to the normality of the data, the pair-sample t-test is used to evaluate the significance of the difference between the pre-test and post-test data of the experimental group.

Table 9. Results of pair-sample t-test to evaluate the significance of the difference between pre-test and post-test data of the experimental group

pair-sample t-test	Freedom degree	Significance level
-4.496	30	0.00

H0 hypothesis refers to the lack of significant difference between the level of understanding of the experimental group students in the pre-test and post-test. According to the results of Table (9), the significance level value is 0.000, which is less than 0.05. Therefore, the H0 hypothesis is false, and there is a significant difference between the students' level of understanding in the pre-test and post-test.

Conclusion

According to Table (6), although the mean of the experimental group in the pre-test and post-test was 1.16, in the control group, it has an increase of 0.65. However, According to Table (8), which is related to the independent t-test, there is no significant difference between the post-test data of the experimental and control groups. In the experimental group compared to the control group, it shows that the independent factor and software variable was not significantly different from the traditional-group method in the students' level of understanding according to APOS theory. It seems that three makeup class sessions were held for the control group, which were mostly based on working groups and solving questions and problems in groups of 6 with the cooperation of the students themselves had a good effect on learning, and there was no significant difference in the post-test results of the control and experimental groups.
In addition, according to Table (9), which is related to the T-test of the two dependent groups, there is a significant difference between the data of the pre-test and post-test of the experimental group and indicates that the independent factor and the problem variable can improve the student's level of understanding based on APOS theory. Decreasing the number of students to at least 1 at zero level (no correct answer) in the experimental group after working with the courseware in the GeoGebra environment is another reason for the positive impact of using this courseware. Therefore, using this software is effective in increasing the students' level of understanding of three-dimensional space concepts $(\mathbb{R}^3)$based on APOS theory.
According to Table (2), in response to the second question of the research, 43.3% of the students in the control group and 42% of the students in the experimental group answered the process level questions correctly in the pre-test. In response to the third question of the research, 20% of the students in the control group and 16.1% of the students in

the experimental group answered the object level questions correctly in the pre-test. In response to the fourth question of the research, 6.7% of the students in the control group and 6.4% of the students in the experimental group answered the schema level questions correctly in the pre-test.

In response to the fifth question of the research, 80% of the students in the control group and 77.4% of the students in the experimental group answered the action level questions correctly in the pre-test. According to Table (4) for the post-test, this value is 86.7% and 96.8%, respectively. Therefore, the percentage of students in the experimental group who answered the pre-test and post-test action level questions correctly is 77.4% and 96.8%, respectively. The 19.4% increase in the experimental group is greater than the 6.7% increase in the control group. These results show that the independent factor and variable of using GeoGebra courseware can be the reason for the better understanding of students in the experimental group than the control group in the post-test action level.

According to Table (2), in response to the sixth question of the research, 43.3% of the students in the control group and 42% of the students in the experimental group answered the process level questions correctly in the pre-test. According to Table (4), this value is 80 and 83.8% for the post-test, respectively. Therefore, the percentage of students in the experimental group who answered the pre-test and post-test level questions correctly is 42 and 83.8%, respectively. The 41.8% increase in the experimental group at this level is greater than the 36.7% increase in the control group. These results show that the independent factor and variable of using GeoGebra courseware can be the reason for the better understanding of students in the experimental group than the control group in the process level at the post-test.

According to Table (2), in response to the seventh question of the research, 20% of the students in the control group and 16.1% of the students in the experimental group answered the object level questions correctly in the pre-test. According to Table (4), this value is 36.7% and 67.7% for the post-test, respectively. Therefore, the percentage of students in the experimental group who answered the pre-test and post-test object-level questions correctly is 16.1% and 67.7%, respectively. The 51.6% increase in the experimental group at this level is greater than the 16.7% increase in the control group. It indicates that the independent factor and variable of using the GeoGebra courseware course can improve and enhance the level of students' understanding at the object level.

In response to the eighth question of the research, according to Table (2), 6.7% of the students in the control group and 6.4% of the experimental group answered the schema level questions correctly at the pre-test. According to Table (4), this value is 10% and 16.1% for the post-test, respectively. Therefore, the percentage of students in the experimental group who answered the pre-test and post-test schema level questions correctly is 6.4% and 16.1%, respectively. The 9.7% increase in the experimental group at this level is greater than the 3.3% increase in the control group. It indicates that the independent factor and variable of using GeoGebra courseware can improve students' understanding at the schema level.

Conclusion

According to the research, which showed an increase in students' level of understanding at all levels of APOS theory and a decrease in zero level (unanswered sheets), it can be concluded that education using GeoGebra courseware can improve the level of students' understanding, which is in line with previous research. In addition, the research of Mohammad Ahmad Al-Khatib (2019) and Yaya Kuzmah (2020) have emphasized the effect of GeoGebra on learning three-dimensional geometry.

Suggestions

The role of technology and computer in better learning mathematics should not be ignored, and some sessions should be dedicated to teaching a workshop on how to use mathematical software. The theoretical foundations of mathematical learning based on mathematical software are suggested to be taught more broadly and seriously in undergraduate and graduate degrees. A few studies have been done on why, how, and where teachers use technology in the classroom. This is especially useful as a guide for implementing object-learning programs, textbooks, software programs, and pre-service and in-service training for teachers and students.

It is recommended to use this software in all stages in schools and Education should pay more attention in its policies. Balancing different mathematical approaches and representations enhances students' insights and abilities. Just using non-visual approaches makes mathematics inflexible and boring, and the excessive use of intuition and visualization also avoids the formal language of mathematics. Therefore, proper use of both approaches will lead to better results in education.

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