NORDHAUS – GADDUM TYPE RESULTS FOR WIENER LIKE INDICES OF GRAPHS

[1]R. BHUVANESHWARI, [2]V. KALADEVI
[1]Research Scholar (FT), PG and Research Department of Mathematics, Bishop Heber College, Trichy-620017, Tamil Nadu, India. Email id: bhuvi950066@gmail.com.
[2]Professor Emeritus, PG and Research Department of Mathematics, Bishop Heber College, Trichy-620017, Tamil Nadu, India. Email id: kaladevi1956@gmail.com.

ABSTRACT
A Nordhaus - Gaddum type result is a lower or upper bound on sum or product of a parameter of a graph and its complement. This concept was introduced in 1956 by Nordhaus E. A., Gaddum J. W. Generalized Wiener like indices such as wiener index, Detour index, Reciprocal- wiener index, Harary- wiener index, Hyper- wiener index, Reciprocal- Detour index, Harary- Detour index and Hyper- Detour index have been studied in graph theory. In this paper, Nordhaus – Gaddum type results of these indices for k-Sun graph and four regular graph are presented.

Keywords: Generalized Wiener like Polynomial, k-sun graph, Nordhaus – Gaddum results.

Introduction 1:

All graphs considered in this paper are finite, simple and connected. For a graph G = (V, E) with vertices u, v ∈ V, the distance between u and v in G, denoted by $d_G(u, v)$, is the length of a shortest (u, v) – path in G. The Wiener index [2,3,4] of G is defined as $W(G)=\frac{1}{2}\sum_{u,v\in V(G)} d_G(u,v)$ with the summation going over all pairs of distinct vertices of G. The above definition can be further generalized in the following way:

$W_\lambda(G)=\frac{1}{2}\sum_{u,v\in V(G)} d_G^\lambda(u,v)$ where

$d_G^\lambda(u,v) = (d_G(u,v))^\lambda$

and λ is any real number.

For particular instances of the invariant λ, W_{-2}, W_{-1} and $\frac{1}{2}W_1 + \frac{1}{2}W_2$ are the so called Harary index [1], reciprocal Wiener index and hyper – wiener index(WW)[5,6].

The detour index of G is defined as $D(G)=\frac{1}{2}\sum_{u,v\in V(G)} D_G(u,v)$ with the summation going over all pairs of distinct vertices of G. The above definition can be further generalized in the following way:

$D_\lambda(G)=\frac{1}{2}\sum_{u,v\in V(G)} D_G^\lambda(u,v)$ where $D(u,v) = (D_G(u,v))^\lambda$ and λ is any real number.

For particular instances of the invariant λ, D_{-2}, D_{-1} and $\frac{1}{2}D_1 + \frac{1}{2}D_2$ are the so called Harary detour index, reciprocal detour index and hyper – detour index(WW). The complement of a graph G, denoted by $\bar{G}$ is the graph with the same vertex set as G, where two vertices in G are adjacent if and only if they are not adjacent in G.

Definition 1.1:

A k – Sun graph (k≥3) is the graph on 2k vertices obtained from a clique $c_1, c_2, \ldots c_k$ on k vertices and an independent set on k vertices. Let V(G) = { $c_1, c_2, \ldots c_k, s_1, s_2, \ldots s_k$} and E(G) = {$s_ic_i$, s_ic_{i+1} ; 1≤i ≤ k} ∪ { s_kc_k, s_kc_1} ∪ {c_ic_j ; 1≤i ≤ k, 1≤j ≤ k, i<j} be the vertex set and edge set of G respectively.

2.2. Generalized Wiener like indices of k – sun graph and its complement graph:

Theorem 2.2.1: Let G be a k – sun graph. Then the generalized wiener polynomial, generalized detour polynomial and the generalized circular polynomial are given by

$$W_\lambda P(G:x) = \left(\frac{k^2+3k}{2}\right)x^{1^\lambda} + (k^2-k)x^{2^\lambda} + (\frac{k^2-3k}{2})x^{3^\lambda}$$

$$D_\lambda P(G:x) = (\frac{k^2-3k}{2})x^{(2k-3)^\lambda} + (k^2-k)x^{(2k-2)^\lambda} + (\frac{k^2+3k}{2})x^{(2k-1)^\lambda}, and$$

$$C_\lambda P(G:x) = (\frac{k^2-3k}{2})x^{(2k-2)^\lambda} + kx^{(2k-1)^\lambda} + k^2x^{2k^\lambda} + kx^{(2k+1)^\lambda} + (\frac{k^2-3k}{2})x^{(2k+2)^\lambda},$$

where k≥4 and λ is any real number.

Proof:
Let G be k – sun graph on 2k vertices, where k≥4 and λ is any real number.
Let V(G) = { $c_1, c_2, \ldots c_k, s_1, s_2, \ldots s_k$} and E(G) = {$s_ic_i$, s_ic_{i+1} ; 1≤i ≤ k} ∪ { s_kc_k, s_kc_1} ∪ {c_ic_j ; 1≤i ≤ k, 1≤j ≤ k, i<j} be the vertex set and edge set of G respectively. The generalized Wiener like polynomial of G is defined as, $W_\lambda P(G:x) = \sum_{u,v\in V(G)} x^{d^\lambda(u.v)}$, for any real number λ.

For the k – sun graph, the generalized wiener polynomial, the generalized detour polynomial and generalized circular polynomial of k – sun graph G are,

$$W_\lambda P(G:x) = \sum_{1\le i\prec j\le k} x^{d^\lambda(c_ic_j)} + \sum_{1\le i\le k} x^{d^\lambda(c_is_i)} + \sum_{1\le i\prec j\le k} x^{d^\lambda(s_is_j)} ----(1)$$

$$D_\lambda P(G:x) = \sum_{1\le i\prec j\le k} x^{D^\lambda(c_ic_j)} + \sum_{1\le i\le k} x^{D^\lambda(c_is_i)} + \sum_{1\le i\prec j\le k} x^{D^\lambda(s_is_j)} ----(2)$$

$$C_\lambda P(G:x) = \sum_{1\le i\prec j\le k} x^{C^\lambda(c_ic_j)} + \sum_{1\le i\le k} x^{C^\lambda(c_is_i)} + \sum_{1\le i\prec j\le k} x^{C^\lambda(s_is_j)} ----(3)$$

The 4-sun graph and the wiener detour matrix are shown in **Figure 1.1** and **Figure 1.2** respectively. **Figure 1.3** shows the circular matrix of the 4 – Sun graph. The wiener-detour matrix and the circular matrix gives the wiener polynomial, the detour polynomial and circular polynomial of the 4 – s-un graph.

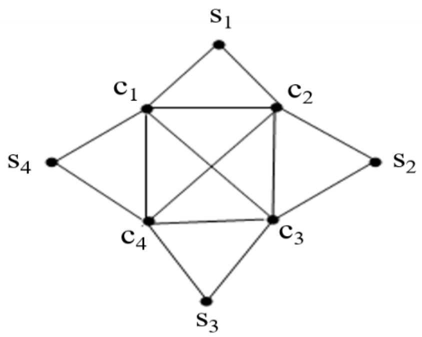

Figure 1.1 4-Sun Graph.

	c_1	c_2	c_3	c_4	s_1	s_2	s_3	s_4
c_1	0	6	5	6	7	6	6	7
c_2	1	0	6	5	7	7	6	6
c_3	1	1	0	6	6	7	7	6
c_4	1	1	1	0	6	6	7	7
s_1	1	1	2	2	0	7	7	7
s_2	2	1	1	2	2	0	7	7
s_3	2	2	1	1	3	2	0	7
s_4	1	2	2	1	2	3	2	0

Figure 1.2 WDM(4-Sun graph)

	c_1	c_2	c_3	c_4	s_1	s_2	s_3	s_4
c_1	0	7	6	7	8	8	8	8
c_2	–	0	7	6	8	8	8	8
c_3	–	–	0	7	8	8	8	8
c_4	–	–	–	0	8	8	8	8
s_1	–	–	–	–	0	9	10	9
s_2	–	–	–	–	–	0	9	10
s_3	–	–	–	–	–	–	0	9
s_4	–	–	–	–	–	–	–	0

Figure 1.3 CM(4-Sun graph)

$$W_\lambda P(G:x) = 14x^{1^\lambda} + 12x^{2^\lambda} + 2x^{3^\lambda}; D_\lambda P(G:x) = 2x^{5^\lambda} + 12x^{6^\lambda} + 14x^{7^\lambda}; C_\lambda P(G:x) = 2x^{6^\lambda} + 4x^{7^\lambda} + 16x^{8^\lambda} + 4x^{9^\lambda} + 2x^{10^\lambda}.$$

The 5-sun graph and the wiener detour matrix are shown in **Figure 1.4** and **Figure 1.5** respectively. **Figure 1.6** shows the circular matrix of the 5 – sun graph. The wiener-detour matrix and the circular matrix gives the wiener polynomial, the detour polynomial and circular polynomial of the 5 – sun graph.

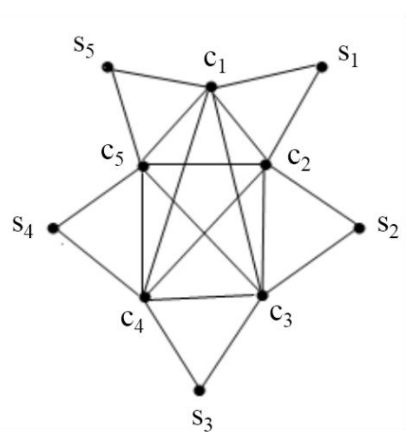

Figure 1.4 5-Sun Graph.

	c_1	c_2	c_3	c_4	c_5	s_1	s_2	s_3	s_4	s_5
c_1	0	8	7	7	8	9	8	8	8	9
c_2	1	0	8	7	7	9	9	8	8	8
c_3	1	1	0	8	7	8	9	9	8	8
c_4	1	1	1	0	8	8	8	9	9	8
c_5	1	1	1	1	0	8	8	8	9	9
s_1	1	1	2	2	2	0	9	9	9	9
s_2	2	1	1	2	2	2	0	9	9	9
s_3	2	2	1	1	2	3	2	0	9	9
s_4	2	2	2	1	1	3	3	2	0	9
s_5	1	2	2	2	1	2	3	3	2	0

Figure 1.5 WDM(5-Sun graph)

	c_1	c_2	c_3	c_4	c_5	s_1	s_2	s_3	s_4	s_5
c_1	0	9	8	8	9	10	10	10	10	10
c_2	–	0	9	8	8	10	10	10	10	10
c_3	–	–	0	9	8	10	10	10	10	10
c_4	–	–	–	0	9	10	10	10	10	10
c_5	–	–	–	–	0	10	10	10	10	10
s_1	–	–	–	–	–	0	11	12	12	11
s_2	–	–	–	–	–	–	0	11	12	12
s_3	–	–	–	–	–	–	–	0	11	12
s_4	–	–	–	–	–	–	–	–	0	11
s_5	–	–	–	–	–	–	–	–	–	0

Figure 1.6 CM(5-Sun graph)

$W_\lambda P(G:x) = 20x^{1^\lambda} + 20x^{2^\lambda} + 5x^{3^\lambda}; D_\lambda P(G:x) = 5x^{7^\lambda} + 20x^{8^\lambda} + 20x^{9^\lambda}; C_\lambda P(G:x) = 5x^{8^\lambda} + 5x^{9^\lambda} + 25x^{10^\lambda} + 5x^{11^\lambda} + 5x^{12^\lambda}$. For k =6, the corresponding wiener polynomial, detour polynomial and polynomial of 6 – sun graph given below,

$W_\lambda P(G:x) = 27x^{1^\lambda} + 30x^{2^\lambda} + 9x^{3^\lambda}; D_\lambda P(G:x) = 9x^{9^\lambda} + 30x^{10^\lambda} + 27x^{11^\lambda}; C_\lambda P(G:x) = 9x^{10^\lambda} + 6x^{11^\lambda} + 36x^{12^\lambda} + 6x^{13^\lambda} + 9x^{14^\lambda}$.

For k =7, the corresponding wiener polynomial, detour polynomial and polynomial of 7 – sun graph given below,

$W_\lambda P(G:x) = 35x^{1^\lambda} + 42x^{2^\lambda} + 14x^{3^\lambda}; D_\lambda P(G:x) = 14x^{11^\lambda} + 42x^{12^\lambda} + 35x^{13^\lambda}; C_\lambda P(G:x) = 14x^{12^\lambda} + 7x^{13^\lambda} + 49x^{14^\lambda} + 7x^{15^\lambda} + 14x^{16^\lambda}$.

For k =8, the corresponding wiener polynomial, detour polynomial and polynomial of 8 – sun graph given below,

$W_\lambda P(G:x) = 44x^{1^\lambda} + 56x^{2^\lambda} + 20x^{3^\lambda}; D_\lambda P(G:x) = 20x^{13^\lambda} + 56x^{14^\lambda} + 44x^{15^\lambda}; C_\lambda P(G:x) = 20x^{14^\lambda} + 8x^{15^\lambda} + 64x^{16^\lambda} + 8x^{17^\lambda} + 20x^{18^\lambda}$.

Hence in general, the generalized wiener polynomial, the detour polynomial and the generalized circular polynomial of k – sun graph G are given by,

$$W_\lambda P(G:x) = \left(\frac{k^2+3k}{2}\right)x^{1^\lambda} + (k^2-k)x^{2^\lambda} + \left(\frac{k^2-3k}{2}\right)x^{3^\lambda}$$

$$D_\lambda P(G:x) = \left(\frac{k^2-3k}{2}\right)x^{(2k-3)^\lambda} + (k^2-k)x^{(2k-2)^\lambda} + \left(\frac{k^2+3k}{2}\right)x^{(2k-1)^\lambda}$$

$$C_\lambda P(G:x) = \left(\frac{k^2-3k}{2}\right)x^{(2k-2)^\lambda} + kx^{(2k-1)^\lambda} + k^2x^{2k^\lambda} + kx^{(2k+1)^\lambda} + \left(\frac{k^2-3k}{2}\right)x^{(2k+2)^\lambda}$$

Corollary 2.2.2:

Let G be a k-sun graph for k≥4. Then, the Wiener index $W_1(G) = 4k^2-5k$,

The Reciprocal-Wiener index $W_{-1}(G) = -[4k^2-5k]$,

The Harary-Wiener index $W_{-2}(G) = -2[4k^2-5k]$,

The Hyper-Wiener index $WW(G) = \frac{1}{2}[12k^2-15k]$.

Corollary 2.2.3:

Let G be a k- sun graph for k≥4. Then

The Detour index $D_1(G) = [4k^3 - 6k^2 + 5k]$, The Reciprocal Detour index $D_{-1}(G) = -[4k^3 - 6k^2 + 5k]$,

The Harary - Detour index $D_{-2}(G) = -2[4k^3 - 6k^2 + 5k]$, The Hyper - Detour index $DD(G) = \frac{1}{2}[12k^3 - 18k^2 + 15k]$.

Theorem 2.2.4:

Let $\bar{G}$ be the complement of k – sun graph G. Then the generalized wiener polynomial and detour polynomial for $\bar{G}$ are respectively given by:

$$W_\lambda P(\bar{G} : x) = \frac{(3k^2 - 5k)}{2} x^{1^\lambda} + \frac{(k^2 + 3k)}{2} x^{2^\lambda} ; D_\lambda P(\bar{G} : x) = k(2k-1) x^{(2k-1)^\lambda}, k \geq 5$$

Proof:

Let G be the k – sun graph on 2k vertices. Let $\bar{G}$ be the complement of k – sun graph G. **Figure 1.7.** shows the complement graph $\bar{G}$ of 5 – sun graph. The wiener detour matrix of the complement of 5 – Sun graph in **Figure 1.8.** gives the wiener polynomial and the detour polynomial of complement of 5 – sun graph. **Figure 1.8.WDM[the complement of 5 – Sun graph]**

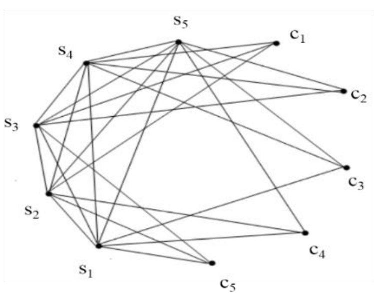

Figure 1.7 $\bar{G}$ (5 – sun graph)

	c_1	c_2	c_3	c_4	c_5	s_1	s_2	s_3	s_4	s_5
c_1	0	9	9	9	9	9	9	9	9	9
c_2	2	0	9	9	9	9	9	9	9	9
c_3	2	2	0	9	9	9	9	9	9	9
c_4	2	2	2	0	9	9	9	9	9	9
c_5	2	2	2	2	0	9	9	9	9	9
s_1	2	1	1	1	2	0	9	9	9	9
s_2	1	2	1	1	1	1	0	9	9	9
s_3	1	2	2	1	1	1	1	0	9	9
s_4	1	1	2	2	1	1	1	1	0	9
s_5	2	1	1	2	2	1	1	1	1	0

Figure 1.8 WDM($\bar{G}$ (5 – sun graph))

$$W_\lambda P(\bar{G} : x) = 25x^{1^\lambda} + 20x^{2^\lambda} ; D_\lambda P(\bar{G} : x) = 45x^{9^\lambda}$$

The wiener detour matrix of the complement of 6 – Sun graph in **Figure 1.9** gives the wiener polynomial and the detour polynomial of complement of 6 – sun graph.

	c_1	c_2	c_3	c_4	c_5	c_6	s_1	s_2	s_3	s_4	s_5	s_6
c_1	0	11	11	11	11	11	11	11	11	11	11	11
c_2	2	0	11	11	11	11	11	11	11	11	11	11
c_3	2	2	0	11	11	11	11	11	11	11	11	11
c_4	2	2	2	0	11	11	11	11	11	11	11	11
c_5	2	2	2	2	0	11	11	11	11	11	11	11
c_6	2	2	2	2	2	0	11	11	11	11	11	11
s_1	2	1	1	1	1	2	0	11	11	11	11	11
s_2	1	2	1	1	1	1	1	0	11	11	11	11
s_3	1	2	2	1	1	1	1	1	0	11	11	11
s_4	1	1	2	2	1	1	1	1	1	0	11	11
s_5	1	1	1	2	2	1	1	1	1	1	0	11
s_6	2	1	1	1	2	2	1	1	1	1	1	0

Figure 1.9 WDM[the complement of 6 – Sun graph]

$W_\lambda P(\bar{G}:x) = 39x^{1^\lambda} + 27x^{2^\lambda}$; $D_\lambda P(\bar{G}:x) = 66x^{11^\lambda}$

For k =7,8,9 the corresponding wiener polynomials and detour polynomial of the complement of k – sun graph given below,

$W_\lambda P(\bar{G}:x) = 56x^{1^\lambda} + 35x^{2^\lambda}$; $W_\lambda P(\bar{G}:x) = 76x^{1^\lambda} + 44x^{2^\lambda}$; $W_\lambda P(\bar{G}:x) = 90x^{1^\lambda} + 54x^{2^\lambda}$

$D_\lambda P(\bar{G}:x) = 91x^{13^\lambda}$; $D_\lambda P(\bar{G}:x) = 120x^{15^\lambda}$; $D_\lambda P(\bar{G}:x) = 153x^{17^\lambda}$

Hence in general, the generalized wiener and detour polynomial of complement of k – sun graph are respectively given by,

$$W_\lambda P(\bar{G}:x) = \frac{(3k^2-5k)}{2}x^{1^\lambda} + \frac{(k^2+3k)}{2}x^{2^\lambda}; D_\lambda P(\bar{G}:x) = k(2k-1)x^{(2k-1)^\lambda}, \text{ k} \geq 5..$$

Corollary 2.2.5:

Let $\bar{G}$ be the complement of k – sun graph G, then

$$\text{The Wiener index } W_1(\bar{G}) = \frac{1}{2}[5k^2+k]; \text{The Reciprocal index } W_{-1}(\bar{G}) = -\frac{1}{2}[5k^2+k];$$

$$\text{The Harary - Wiener index } W_{-2}(\bar{G}) = -\frac{1}{2}[10k^2+2k]; \text{The Hyper - Wiener index } WW(\bar{G}) = \frac{1}{4}[15k^2+3k]$$

Corollary 2.2.6:

Let $\bar{G}$ be the complement of k – sun graph G, then

$$\text{The Detour index } D_1(\bar{G}) = 4k^3-4k^2+k; \text{The Reciprocal - Detour index } D_{-1}(\bar{G}) = -[4k^3-4k^2+k];$$

$$\text{The Harary - Detour index } D_{-2}(\bar{G}) = -2[4k^3-4k^2+k]; \text{The Hyper - Detour index } DD(\bar{G}) = \frac{[12k^3-12k^2+3k]}{2}$$

Result 2.2.7: Nordhaus – Gaddum Equations of k – sun graph.

$$(i)W_1(G)+W_1(\bar{G})=\frac{13k^2-9k}{2};(ii)W_{-1}(G)+W_{-1}(\bar{G})=-\left[\frac{13k^2-9k}{2}\right]$$

$$(iii)W_{-2}(G)+W_{-2}(\bar{G})=-\left[13k^2-9k\right];(iv)WW(G)+WW(\bar{G})=\frac{39k^2-27k}{4}$$

$$(v)D_1(G)+D_1(\bar{G})=[8k^3-10k^2+6k];(vi)D_{-1}(G)+D_{-1}(\bar{G})=-[8k^3-10k^2+6k]$$

$$(vii)D_{-2}(G)+D_{-2}(\bar{G})=-2[8k^3-10k^2+6k];(viii)DD(G)+DD(\bar{G})=\frac{24k^3-30k^2+18k}{2}$$

3.1. Nordhaus – Gaddum Equation for four regular graph:

In this section the generalized wiener polynomial and generalized detour polynomial of four regular graph and complement of four regular graph are presented and also Nordhaus – Gaddum equation for four regular graph is derived.

Algorithm for Four regular graph:

Input : the number of vertices n of a cyclic graph.
Output : the class four regular graph with 2n vertices.
Begin
Step 1: Take a cycle C_n with vertex set $V = \{v_1, v_2,\ldots v_n\}$ and e dge set $E = \{v_iv_{i+1} \cup v_nv_1: 1\leq i\leq(n-1)\}$.
Step 2: For the edge v_iv_{i+1}, $1\leq i\leq(n-1)$ introduce a new vertex u_i and create new edge v_iu_i and $v_{i-1}u_i$.
Step 3: For the edge v_nv_1 introduce a new vertex u_n and create new edges v_nu_n and v_1u_n.
Step 4: From the set of new vertices u_i, create new edges u_iu_{i+1} for $1\leq i\leq(n-1)$ and an edge u_nu_1.
Step 5: The new four regular graph $G(C_n) = (V, E)$ has the vertex set and edge set
$V_G = \{ v_1,v_2,\ldots v_n, u_1, u_2,\ldots u_n \}$
$E_G = \{ u_iv_{i+1},v_nv_1 , v_iu_i , v_{i+1}u_i , v_nu_n , v_1u_n , u_iu_{i+1} , u_nu_1 / 1\leq i\leq(n-1)\}$.

Generalized Wiener like indices of four regular graph and its complement:

Theorem 3.1.1:

Let $G(C_n)$ be a four regular graph. Then the generalized wiener polynomial and the generalized detour polynomial are given by the following expressions:

$$W_\lambda P(G:x)=4nx^{1^\lambda}+4nx^{2^\lambda}+.....+4nx^{(\frac{n-1}{2})^\lambda}+nx^{(\frac{n+1}{2})^\lambda},$$ when n is odd and n≥3.

$$W_\lambda P(G:x)=4nx^{1^\lambda}+4nx^{2^\lambda}+.....+4nx^{(\frac{n-2}{2})^\lambda}+3nx^{(\frac{n}{2})^\lambda},$$ when n is even and n≥4.

$$D_\lambda P(G:x)=\frac{n(n-1)}{2}x^{(n-1)^\lambda}$$

Proof:

Let $G= G(C_n)$ be a four regular graph with 2n vertices. Let $V(G) =\{v_1, v_2,\ldots v_n\}$and edge set
$E = \{ u_iv_{i+1},v_nv_1 , v_iu_i , v_{i+1}u_i , v_nu_n , v_1u_n , u_iu_{i+1} , u_nu_1 / 1\leq i\leq(n-1)\}$.

Case(i): When n is odd.

Figure 1.10. shows the four regular graph $G(C_3)$ and **Figure 1.11.** gives the wiener polynomial and the detour polynomial of four regular graph $G(C_3)$.

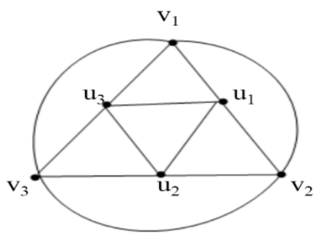

Figure 1.10 G(C3)

	v_1	v_2	v_3	u_1	u_2	u_3
v_1	0	5	5	5	5	5
v_2	1	0	5	5	5	5
v_3	1	1	0	5	5	5
u_1	1	1	2	0	5	5
u_2	2	1	1	1	0	5
u_3	1	2	1	1	1	0

Figure 1.11.WDM[G(C3)]

$$W_\lambda P(G:x) = 12x^{1^\lambda} + 3x^{2^\lambda}; D_\lambda P(G:x) = 15x^{5^\lambda}$$

Figure 1.12. shows the four regular graph $G(C_5)$ and **Figure 1.13.** gives the wiener polynomial and the detour polynomial of four regular graph $G(C_5)$.

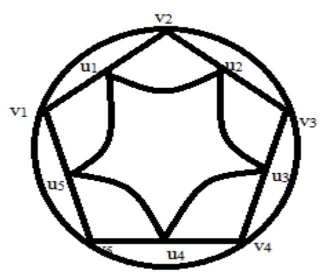

Figure 1.12 G(C5)

	v_1	v_2	v_3	v_4	v_5	u_1	u_2	u_3	u_4	u_5
v_1	0	9	9	9	9	9	9	9	9	9
v_2	1	0	9	9	9	9	9	9	9	9
v_3	2	1	0	9	9	9	9	9	9	9
v_4	2	2	1	0	9	9	9	9	9	9
v_5	1	2	2	1	0	9	9	9	9	9
u_1	1	1	2	3	2	0	9	9	9	9
u_2	2	1	1	2	3	1	0	9	9	9
u_3	3	2	1	1	2	2	1	0	9	9
u_4	2	3	2	1	1	2	2	1	0	9
u_5	1	2	3	2	1	1	2	2	1	0

Figure 1.13.WDM[G(C5)]

$$W_\lambda P(G:x) = 20x^{1^\lambda} + 20x^{2^\lambda} + 5x^{3^\lambda}; D_\lambda P(G:x) = 45x^{9^\lambda}.$$

Hence in general, the generalized wiener polynomial of four regular graph $G(C_n)$ is,

$$W_\lambda P(G:x) = 4nx^{1^\lambda} + 4nx^{2^\lambda} + \ldots + 4nx^{(\frac{n-1}{2})^\lambda} + nx^{(\frac{n+1}{2})^\lambda}$$, when n is odd and $n \geq 3$.

Case (ii): When n is even.

Figure 1.14. shows the four regular graph $G(C_4)$ and **Figure 1.15.** gives the wiener polynomial and the detour polynomial of four regular graph $G(C_4)$.

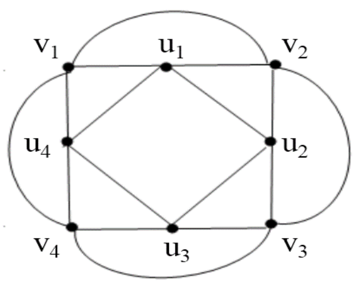

Figure 1.14. G(C4)

	v1	v2	v3	v4	u1	u2	u3	u4
v1	0	7	7	7	7	7	7	7
v2	1	0	7	7	7	7	7	7
v3	2	1	0	7	7	7	7	7
v4	1	2	1	0	7	7	7	7
u1	1	1	2	2	0	7	7	7
u2	2	1	1	2	1	0	7	7
u3	2	2	1	1	2	1	0	7
u4	1	2	2	1	1	2	1	0

Figure 1.15.WM[G(C4)]

$W_{\lambda}P(G:x) = 16x^{1^{\lambda}} + 12x^{2^{\lambda}}; D_{\lambda}P(G:x) = 28x^{7^{\lambda}}$.

Hence in general, the generalized wiener polynomial of four regular graph G(C_n) is

$W_{\lambda}P(G:x) = 4nx^{1^{\lambda}} + 4nx^{2^{\lambda}} + ...+ 4nx^{(\frac{n-1}{2})^{\lambda}} + nx^{(\frac{n+1}{2})^{\lambda}}$, when n is odd and n≥3.

$W_{\lambda}P(G:x) = 4nx^{1^{\lambda}} + 4nx^{2^{\lambda}} + ...+ 4nx^{(\frac{n-2}{2})^{\lambda}} + 3nx^{(\frac{n}{2})^{\lambda}}$, when n is even and n≥4.

and the generalized detour polynomial of four regular graph G(C_n) is,

$$D_{\lambda}P(G:x) = \frac{n(n-1)}{2}x^{(n-1)^{\lambda}}$$

Corollary 3.1.2:

Let G be the four regular graph. Then

The Wiener index $W_1(G) = \frac{n^2(n+1)}{2}$; The Reciprocal - Wiener index $W_{-1}(G) = -\left[\frac{n^2(n+1)}{2}\right]$;

The Harary - Wiener index $W_{-2}(G) = -\left[\frac{n^2(2n+2)}{2}\right]$; The Hyper - Wiener index $WW(G) = \left[\frac{3n^2(n+1)}{4}\right]$.

Corollary 3.1.3:

Let G be the four regular graph, then

The Detour index $D_1(G) = \frac{n(n-1)^2}{2}$; The Reciprocal index $D_{-1}(G) = -\left[\frac{n(n-1)^2}{2}\right]$,

The Harary- Detour index $D_{-2}(G) = -2\left[\frac{n(n-1)^2}{2}\right]$;

The Hyper-Detour index $DD(G) = \left[\frac{3n(n-1)^2}{4}\right].$

Theorem 3.1.4:

Let $\bar{G}$ be the complement of four regular graph G. Then the generalized wiener polynomial and detour polynomial for $\bar{G}$ are respectively given by:

$$W_\lambda P(\bar{G}:x) = (2n^2 - 5n)x^{1^\lambda} + 4nx^{2^\lambda}; D_\lambda P(\bar{G}:x) = (2n^2 - n)x^{(2n-1)^\lambda}.$$

Proof:

Let G be the four regular graph and $\bar{G}$ be the complement of G. **Figure 1.16.** shows the complement graph $\bar{G}$ of four regular graph G(C4). The wiener detour matrix **Figure 1.17.** gives the wiener polynomial and the detour polynomial of complement of four regular graph G(C4).

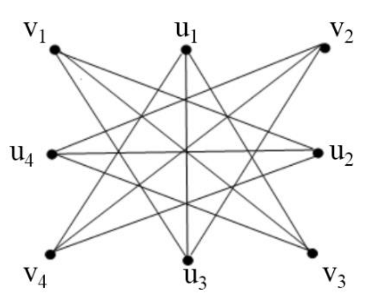

Figure 1.16. $\bar{G}$ of four regular graph G(C4)

	v1	v2	v3	v4	u1	u2	u3	u4
v1	0	7	7	7	7	7	7	7
v2	2	0	7	7	7	7	7	7
v3	1	2	0	7	7	7	7	7
v4	2	1	2	0	7	7	7	7
u1	2	2	1	1	0	7	7	7
u2	1	2	2	1	2	0	7	7
u3	1	1	2	2	1	2	0	7
u4	2	1	1	2	2	1	2	0

Figure 1.17.WDM[$\bar{G}$ (C4)]

$$W_\lambda P(\bar{G}:x) = 12x^{1^\lambda} + 16x^{2^\lambda}; D_\lambda P(\bar{G}:x) = 28x^{7^\lambda}.$$

In similar mannar, when k =5, 6,7,8., the corresponding wiener polynomials of complement graph $\bar{G}$ of four regular graph given below,

$$W_\lambda P(\bar{G}:x) = 25x^{1^\lambda} + 20x^{2^\lambda}; W_\lambda P(\bar{G}:x) = 42x^{1^\lambda} + 24x^{2^\lambda}; W_\lambda P(\bar{G}:x) = 63x^{1^\lambda} + 28x^{2^\lambda};$$

$$W_\lambda P(\bar{G}:x) = 88x^{1^\lambda} + 32x^{2^\lambda}$$

Hence in general, the generalized wiener polynomial of complement of four regular graph,

$$W_\lambda P(\bar{G}:x) = (2n^2 - 5n)x^{1^\lambda} + 4nx^{2^\lambda}$$

In similar mannar, when k =5,6,7,8, the corresponding generalized detour polynomials of complement graph $\bar{G}$ of four regular graph given below,

$D_\lambda P(\bar{G}:x) = 45x^{9^\lambda}; D_\lambda P(\bar{G}:x) = 66x^{11^\lambda}; D_\lambda P(\bar{G}:x) = 91x^{13^\lambda}; D_\lambda P(\bar{G}:x) = 120x^{15^\lambda}$,

Hence in general, the generalized detour polynomial of complement of four regular graph is.

$D_\lambda P(\bar{G}:x) = (2n^2 - n)x^{(2n-1)^\lambda}$.

Corollary 3.1.5:

Let $\bar{G}$ be the complement of G. Then

The Wiener index $W_1(\bar{G}) = [2n^2 + 3n]$; The Reciprocal index $W_{-1}(\bar{G}) = -[2n^2 + 3n]$;

The Harary - Wiener index $W_{-2}(\bar{G}) = -2[2n^2 + 3n]$; The Hyper - Wiener index $WW(\bar{G}) = \frac{1}{2}[6n^2 + 9n]$

Corollary 3.1.6:

Let $\bar{G}$ be the complement of G. Then

The Detour index $D_1(\bar{G}) = 4n^3 - 4n^2 + n$; The Reciprocal - Wiener index $D_{-1}(\bar{G}) = -[4n^3 - 4n^2 + n]$;

The Harary - Detour index $D_{-2}(\bar{G}) = -2[4n^3 - 4n^2 + n]$; The Hyper - Detour index $DD(\bar{G}) = \frac{12n^3 - 12n^2 + 3n}{2}$.

Result 3.1.7: Nordhaus – Gaddum Equations of four regular graph.

$$(i) W_1(G) + W_1(\bar{G}) = \frac{n^3 + 5n^2 + 6n}{2}; (ii) W_{-1}(G) + W_{-1}(\bar{G}) = -\left[\frac{n^3 + 5n^2 + 6n}{2}\right]$$

$$(iii) W_{-2}(G) + W_{-2}(\bar{G}) = -[n^3 + 5n^2 + 6n]; (iv) WW(G) + WW(\bar{G}) = \frac{3n^3 + 15n^2 + 18n}{4}$$

$$(v) D_1(\bar{G}) + D_1(\bar{G}) = \frac{9n^3 - 10n^2 + 3n}{2}; (vi) D_{-1}(\bar{G}) + D_{-1}(\bar{G}) = -\left[\frac{9n^3 - 10n^2 + 3n}{2}\right];$$

$$(vii) D_{-2}(\bar{G}) + D_{-2}(\bar{G}) = -[9n^3 - 10n^2 + 3n]; (viii) DD(\bar{G}) + DD(\bar{G}) = \frac{27n^3 - 30n^2 + 9n}{4}.$$

Conclusions:

In 1956, Nordhaus E. A., Gaddum J. W. [7] introduced the bounds involving the chromatic number χ(G) of a graph G and its complement. Many authors studied [8, 9] Nordhaus-Gaddum bounds for domination number, connected domination number, total domination number and also there has been many publications on Nordhaus-Gaddum type results for indices like Gutman wiener index, Steiner index, Krichhoff index. This paper deals with Nordhaus – Gaddum equations for wiener like indices to k – sun graph four regular graph.

References:

[1] Diudea M. V., Indices of reciprocal properties on Harary indices, J. Chem.Inf. Comput. Sci. 37(1997), pp:292-299.

[2] Gutman I., A property of the Wiener number and its modifications, Indian J. Chem., 36A(1997), pp:128-132.

[3] Gutman I., Dobrynin A.A., Klavzar S., Pavlovic L., Wiener type invariants of tree and their relation, Bull Inst. Combin. Appl. 40(2004), pp:23-30.

[4] Joshi A., Baskar babujee J., Wiener Polynomial for Graphs with Cycle, Proceedings of the International Conference on Mathematics and Computer Science, (2008) pp:119-125

[5] Klavzar S., Zigert P., Gutman I., An algorithm for the calculation of the hyper – Wiener index of benzenoid hydrocarbons, Compt. Chem. 24(2000), pp:229-233

[6] Lukovits I., A note on a formula for the hyper – Wiener of some trees, J. Chem. Inf. Comput. Sci. 34(1994), pp:1079-1081.

[7] Nordhaus E. A., Gaddum J.W., On complementary graphs, Amer. Math. Monthly 63(1956), pp: 175-177.

[8] Payan C., Xuong N. H., Domination-balanced graphs, J. Graph Theory 6 (1982), pp: 23-32.

[9] Plesnik J., Bounds on chromatic numbers of multiple factors of complete graph, J. Graph Theory 2 (1978), pp: 9-17.